BME 200
Lesson 4
Sep 3, 2020

By the end of the class students should be able to:

- Define half-life, rate constant, lumped model
- Use the exponential decay formula to calculate concentration over time
- Determine half-life from the exponential decay formula

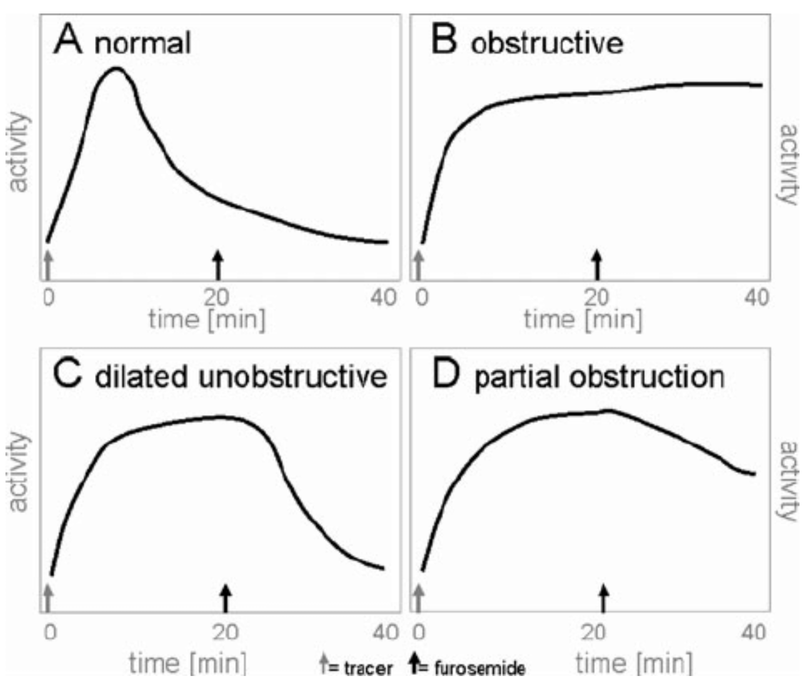
I. Next steps
   A. Two new concepts:
      1. Tissue compartments
      2. Exponentials
   B. Tissue Compartments
      1. Last time we practiced drawing control volumes around systems
      2. CVs can also be an **abstract** representation of a system
      3. For example, I could represent the whole human body like this:
      4. What does this mean?
         a) I have used a rectangle to represent an entire human being.
         b) I made lots of simplifying assumptions.
            (1) No organs
            (2) Water is well-mixed throughout the volume, etc.
         c) <u>Basically:</u> I don't care what's going on inside, all I care about is that there is a body and I have stuff going in and stuff coming out
         d) This is called a **lumped model** because I have lumped everything together into a single block
      5. This is still a CV because it has a boundary and I have mass going in and out
      6. I could use this setup to model all sorts of things.
         a) The composition of gases during inhalation and exhalation
         b) A water balance to compare water going in vs water going out
            (1) Note: I don't specify the different in and out routes, but I could

c) A drug going in and its byproducts coming out
d) And so on

C. Exponentials
1. Last time we thought about systems where we had mass coming in and going out
2. It was at a constant rate
3. But what if the amount going out depends on the amount that's already in there?
4. For example, think about water flowing out of a large container:
5. The water is flowing out because of the hydrostatic pressure in the container; as the level goes down, the water flows out slower, and slower and slower because the pressure is constantly decreasing
6. In other words, the rate of mass out depends on the amount of mass inside the container
7. Here's how I would write this as a mass balance:
8. Now things are a littler harder than last class, because I have a variable and a derivative of the variable in the equation
9. This is a differential equation and you need to use math that some of you haven't had yet, so here are the steps:

10. I don't expect you to derive this equation. Just know the result.
11. Notice a couple of things
    a) M_o is the amount of mass you start with
    b) k is called the **rate constant** and determines how fast the water level drops (k has units of sec^{-1}):

12. What does this have to do with BME? Many applications; two examples:
    a) How fast drugs are metabolized or cleared out of the body (pharmacokinetics)
    b) Tracers - how fast a dye gets cleared out of circulation helps doctors figure out how well your kidneys are working (called clearance rate)
        (1) diuretic renography



From: Gimpel, Charlotte, et al. "Complications and long-term outcome of primary obstructive megaureter in childhood." Pediatric nephrology 25.9 (2010): 1679-1686.

Student Exercise 1

You inject some tracer into a patient. You forget to measure how much mass you started with, but you know the rate constant is $k = 0.3\,\mathrm{hr}^{-1}$. You also know that after 20 min the mass of tracer in the body is 350 mg. How much mass did you inject? (Be sure to pay attention to units).

13. What if we want to know how long it will take to get to a certain amount of water left in the container? In other words, how long will it take to get to 10% of the starting mass? We can rearrange the formula like this:

14. If we use this equation to figure out how long it will take to get 1/2 of the mass, or in other words $\frac{m(t)}{m_o} = 0.5$, we are calculating the **half-life.**
15. But, you can can solve for any amount or time

Student Exercise 2

A second patient arrives at the clinic. You inject some tracer into this new patient and discover that the half life is $t_{\frac{1}{2}} = 2.31\ \text{hr}$. What is the tracer rate constant?