BME 200
Lesson 18
Oct 22, 2020

By the end of the course students should be able to:
- Perform hypothesis testing for comparing two independent means

I. Sample versus population
   A. Last time we averaged all heights of students and got a curve that looks like this:

   B. Then, we predicted the likelihood of a random student having a certain height
   C. We used single students as an example last time, but we could also pick 3 or more students, and average their numbers, and predict what the odds are that they would fall into a certain height range
   D. If we pick 3 students, and average their numbers, this is called a **sample.** We say that it's a sample with $n = 3$.
   E. Imagine an experiment:
      1. We randomly sample 3 students and average their heights. Their mean will fall somewhere on the curve
      2. Repeat step 1 a few more times

   F. What do we notice?
      1. It is more likely that the sample means will be near the population mean than not. Very likely near the middle and much less likely out on the tails

G. Now, let's do an experiment. Say I come to class and I say that I took a sample of BME 200 students, averaged it together, and got this. Do you believe me?

H. Put another way, if I give you a sample, can you tell me if it's from one population versus some other population?

I. Here's a more practical example: Let's say I'm developing a new blood pressure drug. I give 10 people the drug and 10 people the placebo. I want to know if the drug lowers blood pressure. What I'm really trying to figure out is if the people on the blood pressure medication are a sample that comes from a different population than those on the placebo.
    1. If you start to look at the world in these terms, you can apply statistical testing to almost anything.

J. Comparing two means - this is the whole idea behind **statistical testing**

K. There are several types of tests when you compare means. The one we're going to do is called "hypothesis testing the mean"

II. Hypothesis testing
- A. Here are the steps
    1. **Formulate a hypothesis.** You will always know the mean of one population, called μ_0. What you are testing is if your sample comes from a different population with a different mean, μ. There are two possibilities:

        - $\mu \geq \mu_0$
        - $\mu \leq \mu_0$

        Choose the one that is the opposite of what you want to test. For example, if you want to test that your measured sample comes from a population with a higher mean, then you would choose $\mu \leq \mu_0$. You write this "$H_0 : \mu \leq \mu_0$". This is called the *null hypothesis,* H_0. The null hypothesis is typically the hypothesis that represents no change.
    2. **Select test statistic.** We will be using t for the test statistic. This is known as a Student t-test and we will look up values from the t-table.
    3. **Establish the significance level.** This value determines how confident you want to be in your prediction. The most common value is 5%, written ($p < 0.05$). This percentage means that there is less than a 5% chance that you are incorrect about your prediction. Or, in other words, there is less than a 5% chance that Pr[reject $H_o|H_o$ true].

        Once you select your percentage, write it down like this: $\alpha = 0.05$. Then, look up the t value from the table for your alpha value. You will need to use this formula: $df =$

n-1 to get the correct df value. For example, assume $n = 25$ and $\alpha = 0.05$. Then $df = 25 - 1 = 24$ so $t_{.05} = 1.711$.

4. **Sketch a curve for your problem.** This will help you visualize the problem.
5. **Compute the value for *t*.** Compute the t value specific to your problem. Use this formula:

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}}$$

   Here, $\bar{X}$ is the population mean you are testing against. The value μ_0 is the mean of your test sample, s is the sample standard deviation, and n is the sample size.

6. **Make the decision.** Determine if your calculated t value falls within the rejection region. If so, reject H_0. If not, you must not reject H_0.

III. Example.

You work at a medical device company that makes hip implants. The company currently has a product on the market, AccuHip®, but marketing wants to start selling an "improved" hip design, the AccuHip Xtreme®. Your task is to figure out if the AccuHip Xtreme lasts longer than the AccuHip. The company tells you that the AccuHip lasts an average of $\mu_0 = 501{,}000$ cycles before failure. Since the AccuHip Xtreme is still a prototype, you can only measure 25 units, but you find that it lasts for an average of $\bar{X} = 519{,}500$ cycles before failure. You know that the sample standard deviation is 48,732. Should the company adopt the AccuHip Xtreme design?

IV. Student example.

You work at a startup that is developing a new drug, Hypotensivil®, to treat hypertension (high blood pressure). Your job is to analyze the data from the clinical trials and make a recommendation to management about whether to proceed in marketing the drug. Management only wants to market the drug if it results in a reduction of systolic blood pressure below $\mu_0 = 160$ mmHg for hypertensive patients. In the clinical trials, these were the results of test patients who received Hypotensivil®: n = 81 and the average systolic blood pressure was 149 mmHg with a sample standard deviation of 29 mmHg.

The company wants to be certain that the drug works, and suggests that you use a confidence level of 1%. Should the firm proceed with marketing the drug?

http://statstutorstl.blogspot.com/2010/07/t-table.html

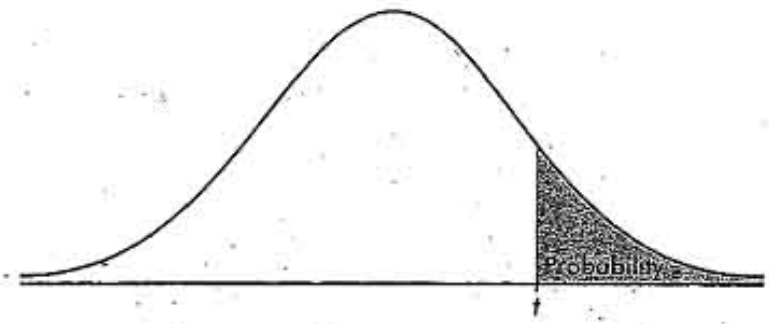

TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level *C*											