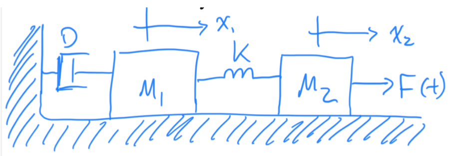
B. Example 2. Write the state-space representation of the system below. Assume there is no output.



STEP 1: WRITE EQUATIONS

LIKE BEFORE, BUT WITH THIS CHANGE: $s^2X_1(s) \Rightarrow \ddot{x}_1$

$sX_1(s) \Rightarrow \dot{x}_1$

$X_1(s) \Rightarrow x_1$

TWO LINEAR MOTIONS, SO TWO EQUATIONS:

FOR M_1: $M_1\ddot{x}_1 + D\dot{x}_1 + Kx_1 - Kx_2 = 0$

FOR M_2: $M_2\ddot{x}_2 + Kx_2 - Kx_1 = F$

STEP 2: RE-WRITE EQUATIONS IN TERMS OF $\dot{x}$ AND $\dot{v}$ + DIVIDE THROUGH BY THE MASS

$\dot{x}_1 = v_1$ (BECAUSE IN PHYSICS VELOCITY = DERIVATIVE OF POS)

$v = \frac{dx}{dt}$

$v = \dot{x}$

$$\dot{v}_1 = -\frac{Dv_1}{M_1} - \frac{Kx_1}{M_1} + \frac{Kx_2}{M_1}$$

$\dot{v}_1 = \ddot{x}_1$

= ACCELERATION

$\dot{x}_2 = v_2$

$$\dot{v}_2 = \frac{-Kx_2}{M_2} + \frac{Kx_1}{M_2} + \frac{F}{M_2}$$

STEP 3: WRITE IN MATRIX FORMAT

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} F$$

$$\dot{x} = A \qquad x + B \quad u$$

THERE IS NO OUTPUT EQUATION