

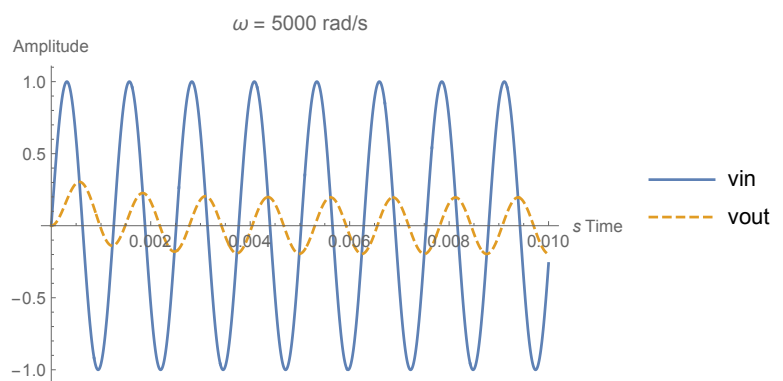
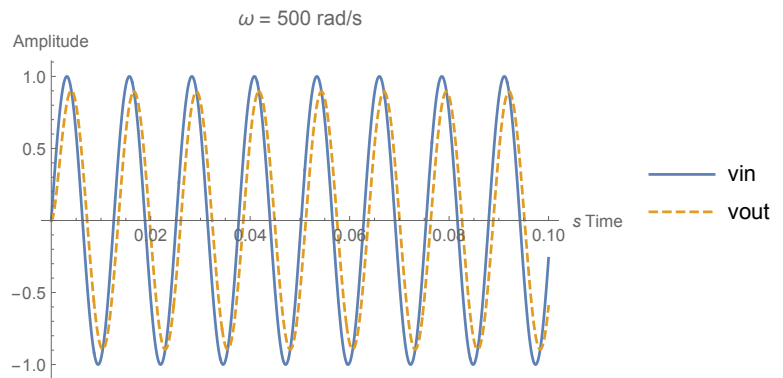
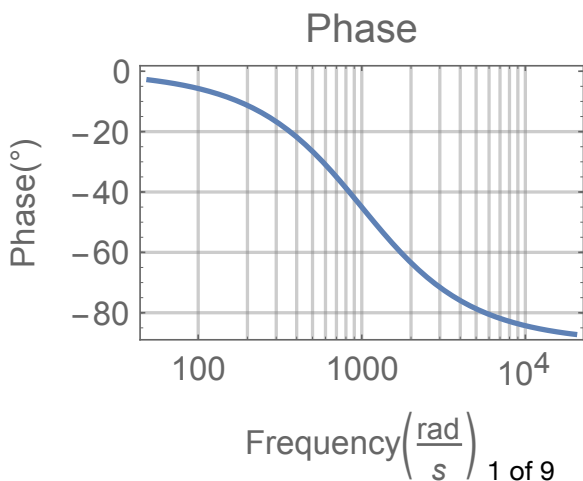
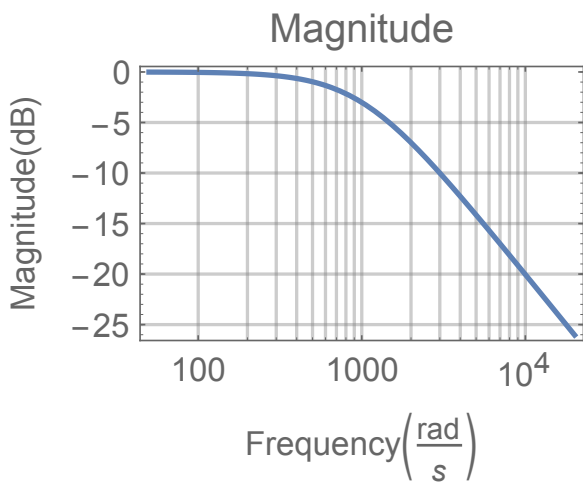
## Lesson 12

### BME 444 - Control Systems

By the end of this lecture students will be able to:

- Calculate the cutoff frequency, time constant, and DC gain of a first-order system
- Calculate bandwidth of first and second-order systems
- Explain effect of  $\zeta$  on the frequency response of second-order systems
- Design filters with specific cutoff frequencies
- Explain what a filter does
- Calculate the location of the peak in the frequency response of a second-order system

- I. What does the frequency response tell us?
  - A. Previously we looked at unit and impulse inputs to systems and plotted their output
  - B. The frequency response of the RC circuit from last time is shown below.



- C. Getting precise values from phase and magnitude plots:

```
w=500;  
[mag, phase]=bode(G, w)
```

```
mag = 0.8944
```

```
phase = -26.5651
```

```
w=5000;  
[mag, phase]=bode(G, w)
```

```
mag = 0.1961
```

```
phase = -78.6901
```

- D. Important: The mag and phase values that bode() returns are magnitude (not dB) and degrees (not radians).
- E. You can also create a phase magnitude with state-space representations after you define the tf

## II. Phase and magnitude plots of first-order systems

- A. When we first learned about first-order systems this is the equation we used (assuming  $y_0 = 0$ ):

$$y(s) = \frac{y_{ss}}{\tau s + 1}$$

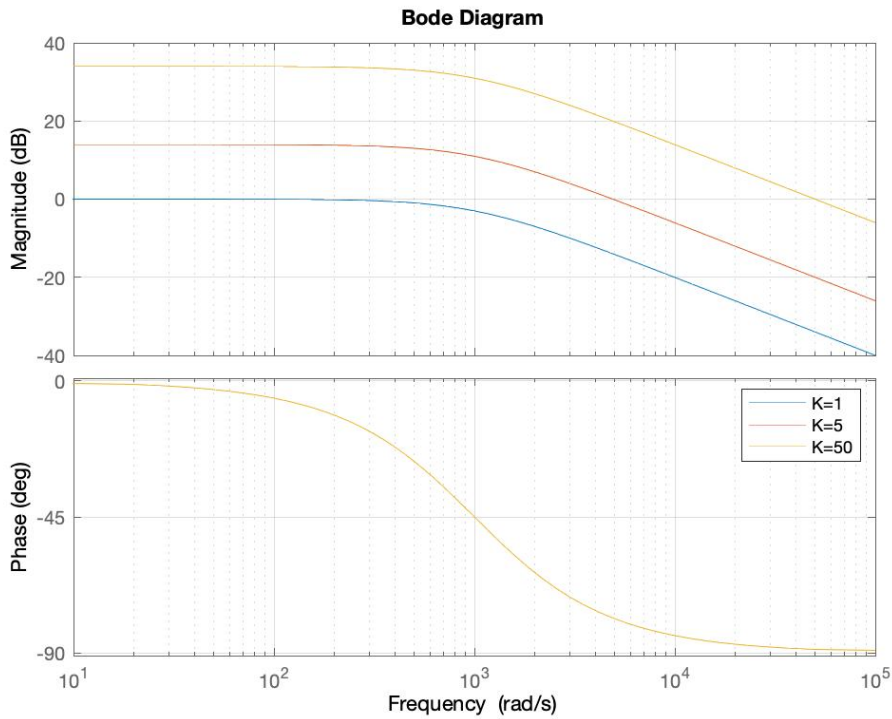
- B. In this equation,  $\tau$  is called the time constant and  $y_{ss}$  is the steady-state value, but more often it's called the *DC gain* and represented with a  $K$ .

$$y(s) = \frac{K}{\tau s + 1}$$

- C. **Important:** note the “+1” in the denominator. You must manipulate the fraction until the denominator contains “+1” for the other two coefficients to be  $\tau$  and  $K$ .

D. Let's see what effect the DC gain has on the Bode plots.

$$\text{Assume } G(s) = \frac{K}{0.001s + 1}$$

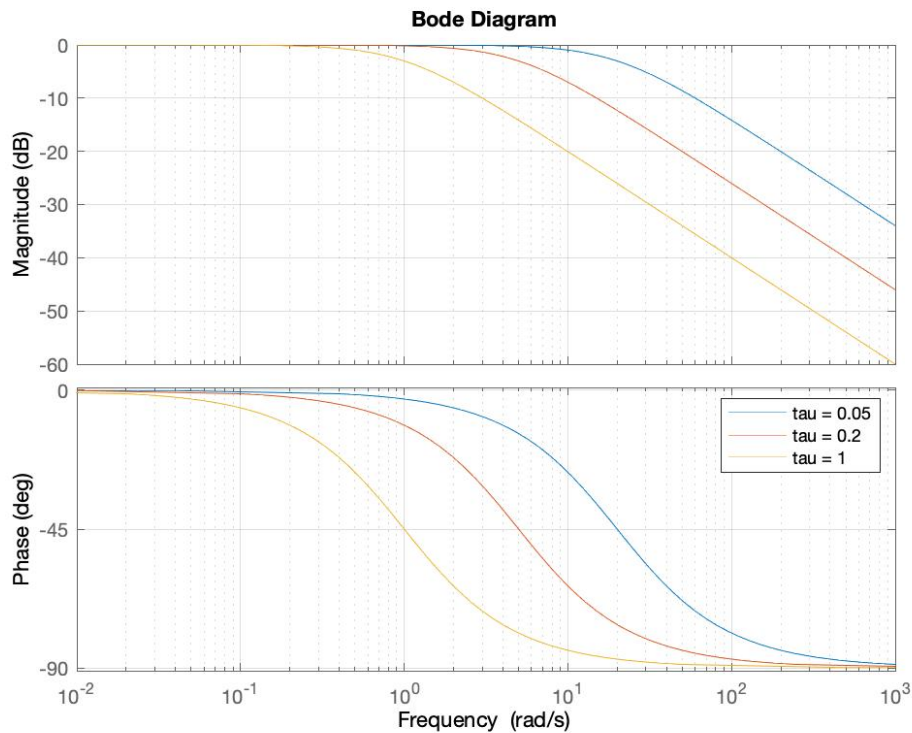


E. Main points for first-order systems:

1. Increasing  $K$  shifts magnitude curve higher
2. Phase plot is not affected by the  $K$
3. Phase angle always starts at  $0^\circ$  and approaches  $-90^\circ$  at very high frequencies

F. Next, let's see what effect changing  $\tau$  has on the frequency response.

$$\text{Assume } G(s) = \frac{1}{\tau s + 1}$$



G. Main points:

1. Phase angle still starts at  $0^\circ$  and ends at  $-90^\circ$
2.  $\tau$  changes the frequency of a phase angle of  $-45^\circ$

H. Important points about first-order systems in general

1. The frequency where the phase angle is  $-45^\circ$  is known as the cutoff frequency. At this frequency, the magnitude is -3dB less than the value at 0 rad/sec.
2. The magnitude up to the corner frequency is flat; the magnitude past the corner frequency is -20 dB/decade. The point where these two intersect is the cutoff frequency.
3. Calculate the cutoff frequency with  $\omega_c = 1/\tau$ . This is in radians!

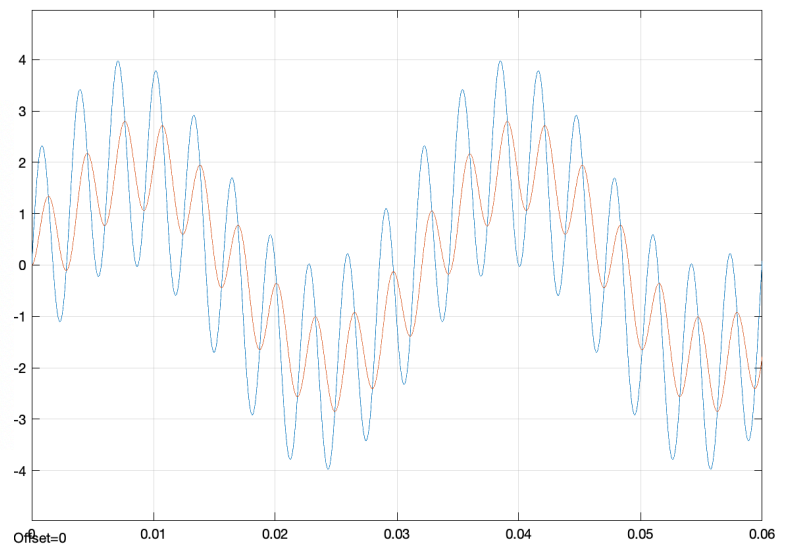
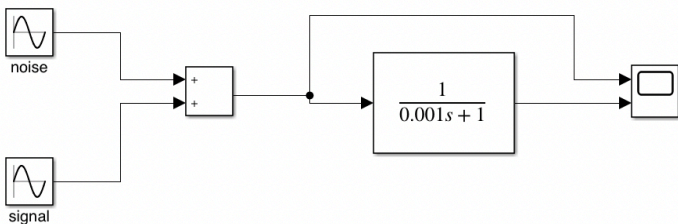
I. **Student exercise 1.** Given the following transfer function, calculate the DC gain, time constant, and cutoff frequency.

$$G(s) = \frac{4}{s + 8}$$

- J. Student exercise 2.** What DC gain does a first-order system have if the frequency response shows a magnitude of 25 dB for frequencies near 0 rad/sec? Assume a  $\tau = 0.001$ .

### III. Cutoff frequency

- A. The cutoff frequency is most often used in circuits called filters that remove certain frequencies from a signal and retain others.
- B. The cutoff is the threshold.
- C. **Example 1.** Assume the RC circuit that we have been working with. And that it has a transfer function of  $G(s) = \frac{1}{0.001s + 1}$ . Assume two inputs:  $v_{noise}(t) = 2 \sin(2000t)$  and  $v_{in}(t) = 2 \sin(200t)$ .



- D. **Student exercise 3.** What amplitudes should  $v_{noise}(t)$  and  $v_{in}(t)$  have at the output? At what frequency will  $v_{noise}(t)$  be 3 dB smaller in amplitude than  $v_{in}(t)$ ?

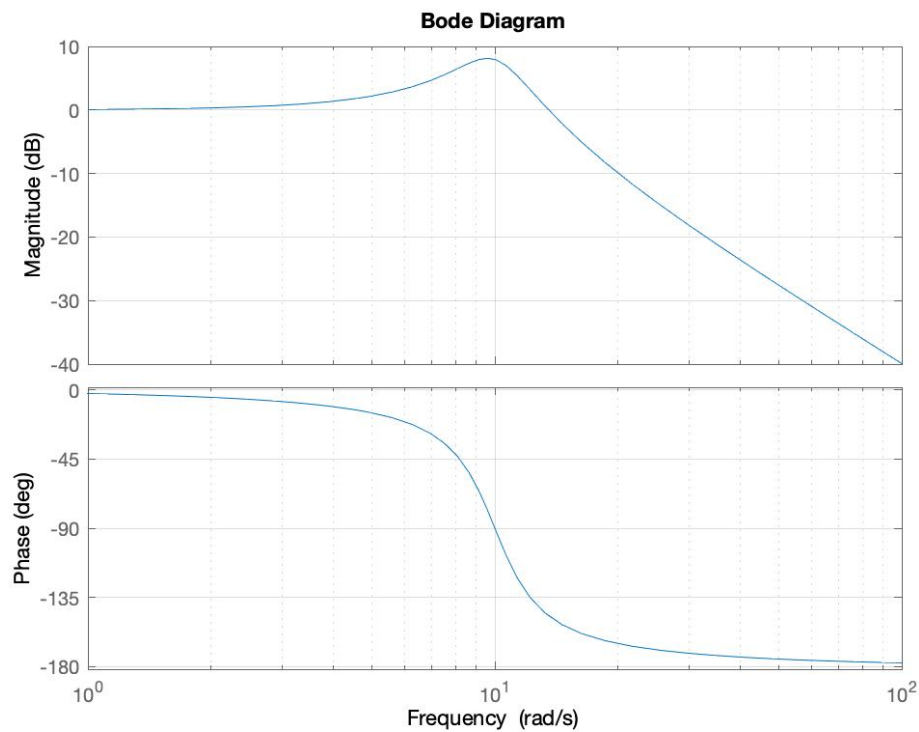
#### IV. Phase and magnitude plots of second-order systems

- A. Consider the standard form of a second-order system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- B. Assume a second-order system, here is what the bode plot looks like

$$G(s) = \frac{100}{s^2 + 4s + 100}$$



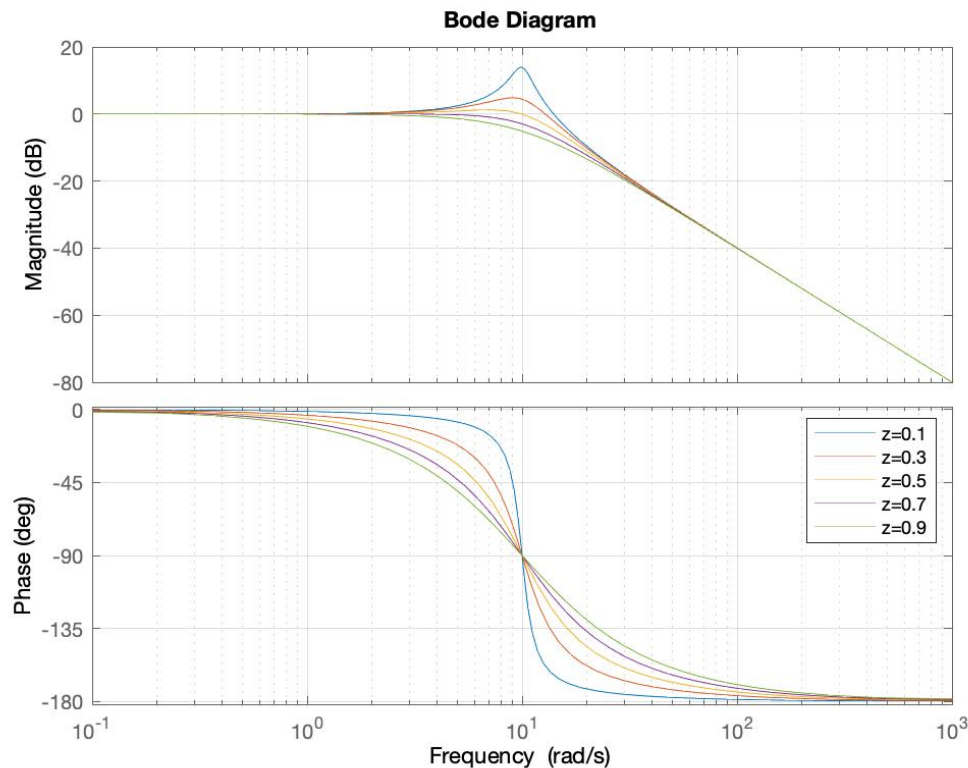
C. Key points:

1. Phase angle ranges from  $0^\circ$  to  $180^\circ$
2. 2nd order systems can have a peak in them located at the damped frequency

$$\omega_d = \omega_n \sqrt{1 - 2\zeta^2}$$

3. On magnitude graph, horizontal at low frequencies and then  $-40$  dB/decade over rest

D. Effect of  $\zeta$  on bode plot



1. Main points

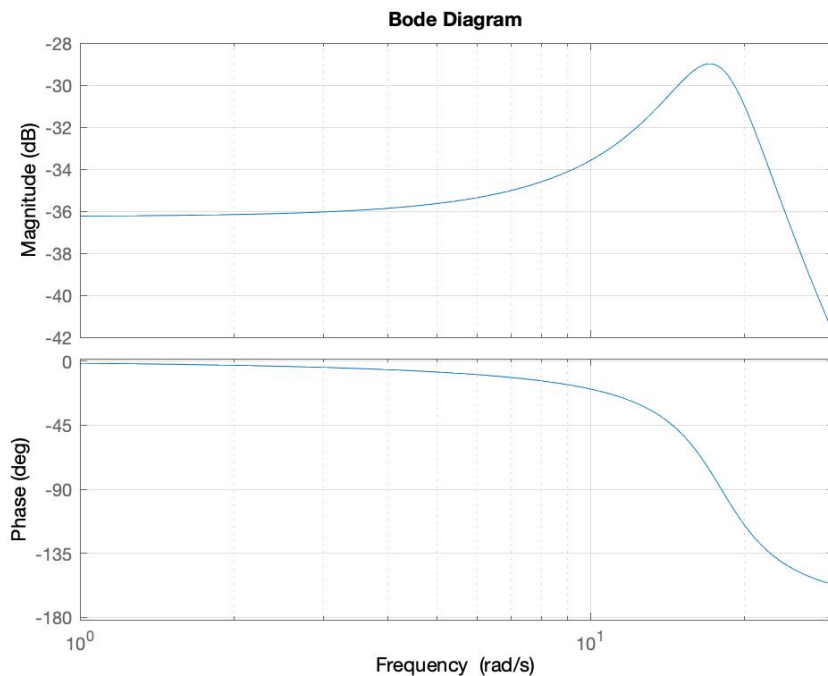
- a) As  $\zeta$  increases peak magnitude decreases
- b) Phase angle transition becomes sharper at lower  $\zeta$
- c) Phase angle of  $90^\circ$  is the corner frequency  $\omega_c$
- d) At  $\zeta > 0.707$  there is no resonant peak

**E. Student exercise 4.** Consider the transfer function of the rotational system below. Generate a bode plot of the transfer function. Compute the frequency of the peak in the bode plot. Determine the output if the input is  $T_{in}(t) = 1.5 \sin(18t)$  N-m.

$$G(s) = \frac{\theta(s)}{T_{in}(s)} = \frac{1}{0.2s^2 + 1.6s + 65}$$

## V. Bandwidth

- A. The bandwidth is the range of frequencies that a system will stay within 3 dB of the DC gain and up to the cutoff frequency
- B. You can visually determine this from a plot
- C. **Student exercise 5.** Estimate the bandwidth of the system response plotted below.



- D. Or you can use the matlab command `bandwidth(tf)`. Answer will be in rad



- E. **Student exercise 6.** Compute the bandwidth of the system represented by the transfer function shown below

$$G(s) = \frac{5}{s^2 + 8s + 325}$$