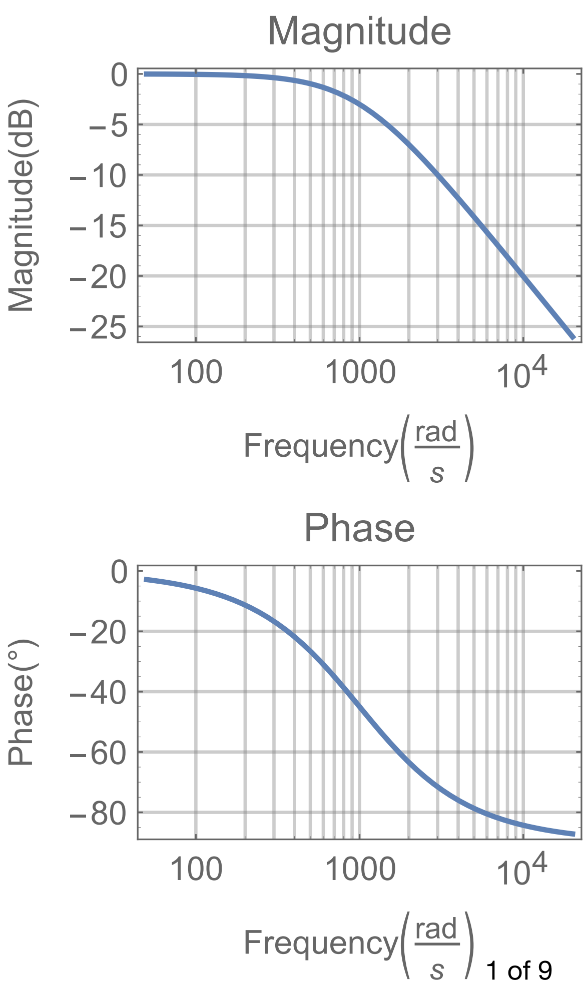
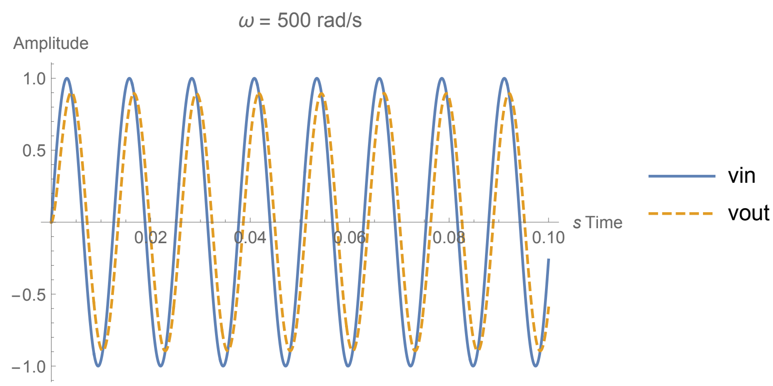
**Lesson 12**
**BME 444 - Control Systems**

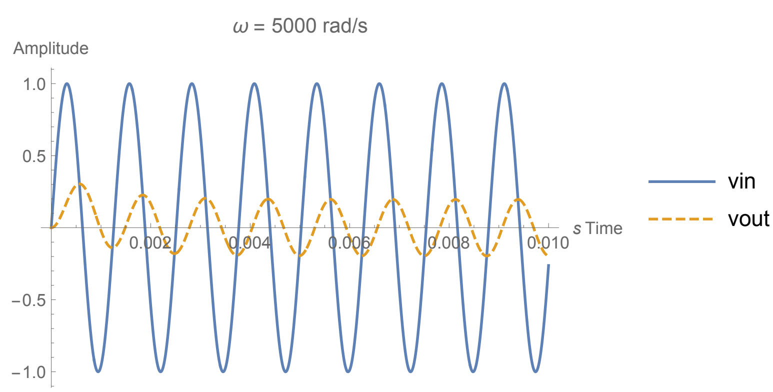
By the end of this lecture students will be able to:

- Calculate the cutoff frequency, time constant, and DC gain of a first-order system
- Calculate bandwidth of first and second-order systems
- Explain effect of $\zeta$ on the frequency response of second-order systems
- Design filters with specific cutoff frequencies
- Explain what a filter does
- Calculate the location of the peak in the frequency response of a second-order system

I. What does the frequency response tell us?
   A. Previously we looked at unit and impulse inputs to systems and plotted their output
   B. The frequency response of the RC circuit from last time is shown below.



C. Getting precise values from phase and magnitude plots:

```
w=500;
[mag,phase]=bode(G,w)

  mag = 0.8944

  phase = -26.5651

w=5000;
[mag,phase]=bode(G,w)

  mag = 0.1961

  phase = -78.6901
```

D. Important: The mag and phase values that bode() returns are magnitude (not dB) and degrees (not radians).
E. You can also create a phase magnitude with state-space representations after you define the tf

II. Phase and magnitude plots of first-order systems
A. When we first learned about first-order systems this is the equation we used (assuming $y_0 = 0$):

$$y(s) = \frac{y_{ss}}{\tau s + 1}$$

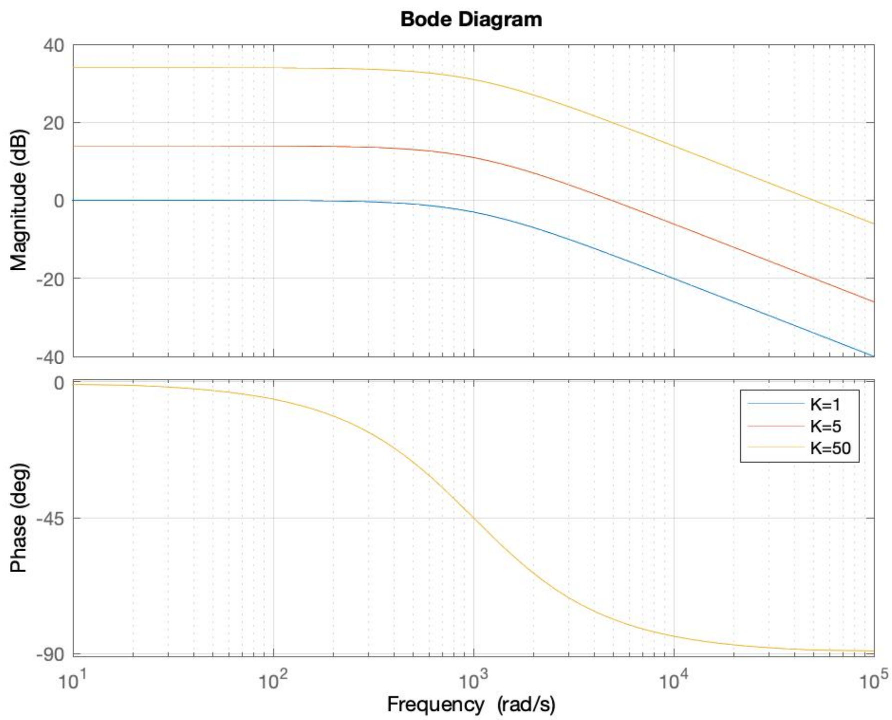
B. In this equation, $\tau$ is called the time constant and $y_{ss}$ is the steady-state value, but more often it's called the *DC gain* and represented with a $K$.

$$y(s) = \frac{K}{\tau s + 1}$$

C. **Important:** note the "+1" in the denominator. You must manipulate the fraction until the denominator contains "+1" for the other two coefficients to be $\tau$ and $K$.

D. Let's see what effect the DC gain has on the Bode plots.

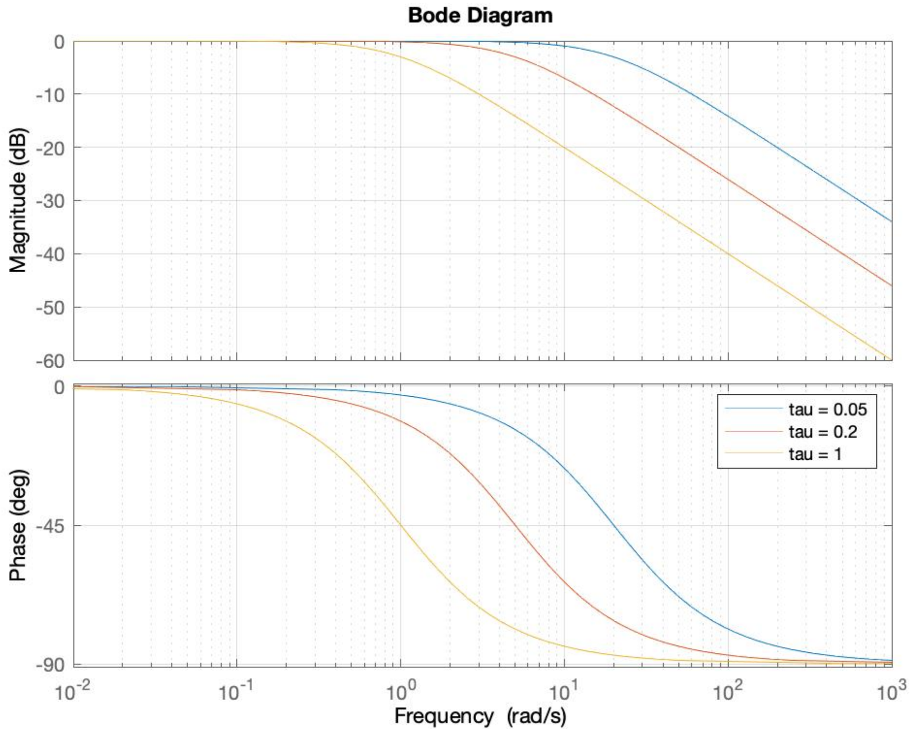
$$\text{Assume } G(s) = \frac{K}{0.001s + 1}$$




E. Main points for first-order systems:
   1. Increasing $K$ shifts magnitude curve higher
   2. Phase plot is not affected by the $K$
   3. Phase angle always starts at 0º and approaches -90º at very high frequencies

F. Next, let's see what effect changing $\tau$ has on the frequency response.

$$\text{Assume } G(s) = \frac{1}{\tau s + 1}$$




G. Main points:
   1. Phase angle still starts at 0º and ends at -90º
   2. $\tau$ changes the frequency of a phase angle of -45º
H. Important points about first-order systems in general
   1. The frequency where the phase angle is -45º is known as the cutoff frequency. At this frequency, the magnitude is - 3dB less than the value at 0 rad/sec.
   2. The magnitude up to the corner frequency is flat; the magnitude past the corner frequency is -20 dB/decade. The point where these two intersect is the cutoff frequency.
   3. Calculate the cutoff frequency with $\omega_c = 1/\tau$. This is in radians!

I. **Student exercise 1.** Given the following transfer function, calculate the DC gain, time constant, and cutoff frequency.

$$G(s) = \frac{4}{s+8}$$

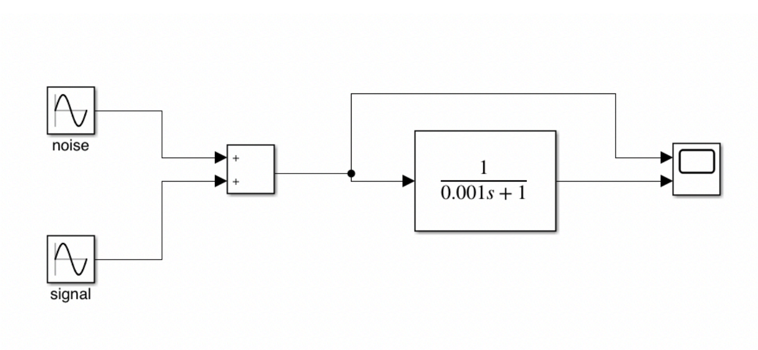
J. **Student exercise 2.** What DC gain does a first-order system have if the frequency response shows a magnitude of 25 dB for frequencies near 0 rad/sec? Assume a $\tau = 0.001$.

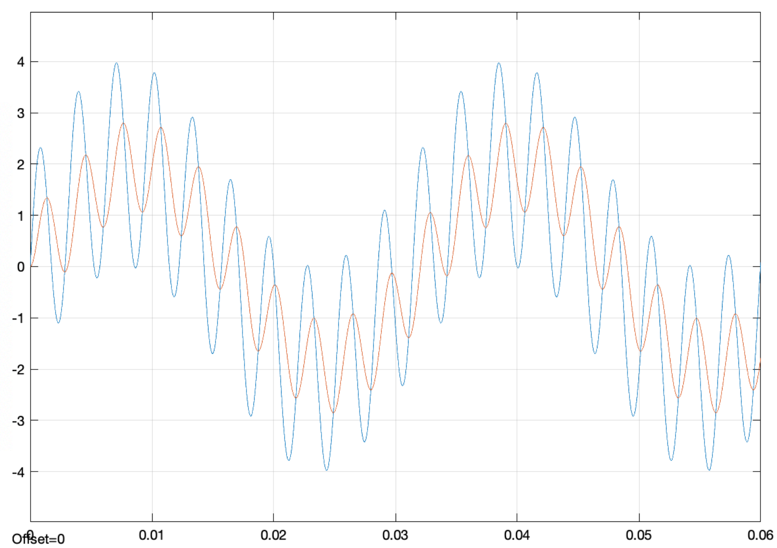
III. Cutoff frequency

A. The cutoff frequency is most often used in circuits called filters that remove certain frequencies from a signal and retain others.

B. The cutoff is the threshold.

C. **Example 1.** Assume the RC circuit that we have been working with. And that it has a transfer function of $G(s) = \dfrac{1}{0.001s + 1}$. Assume two inputs: $v_{noise}(t) = 2\sin(2000t)$ and $v_{in}(t) = 2\sin(200t)$.



D. **Student exercise 3.** What amplitudes should $v_{noise}(t)$ and $v_{in}(t)$ have at the output? At what frequency will $v_{noise}(t)$ be 3 dB smaller in amplitude than $v_{in}(t)$?

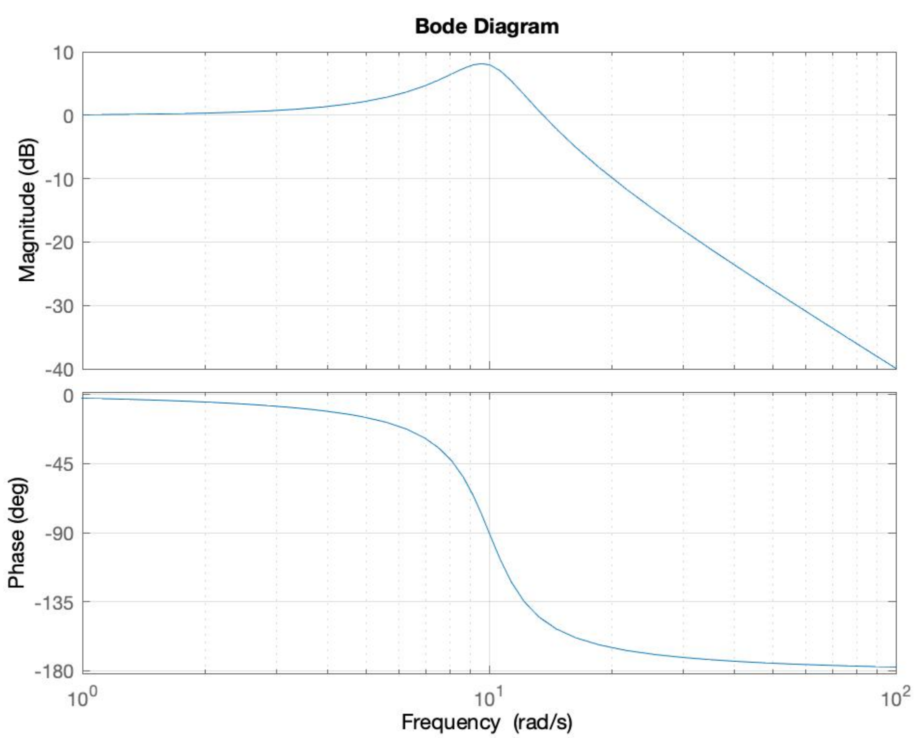
IV. Phase and magnitude plots of second-order systems

A. Consider the standard form of a second-order system

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

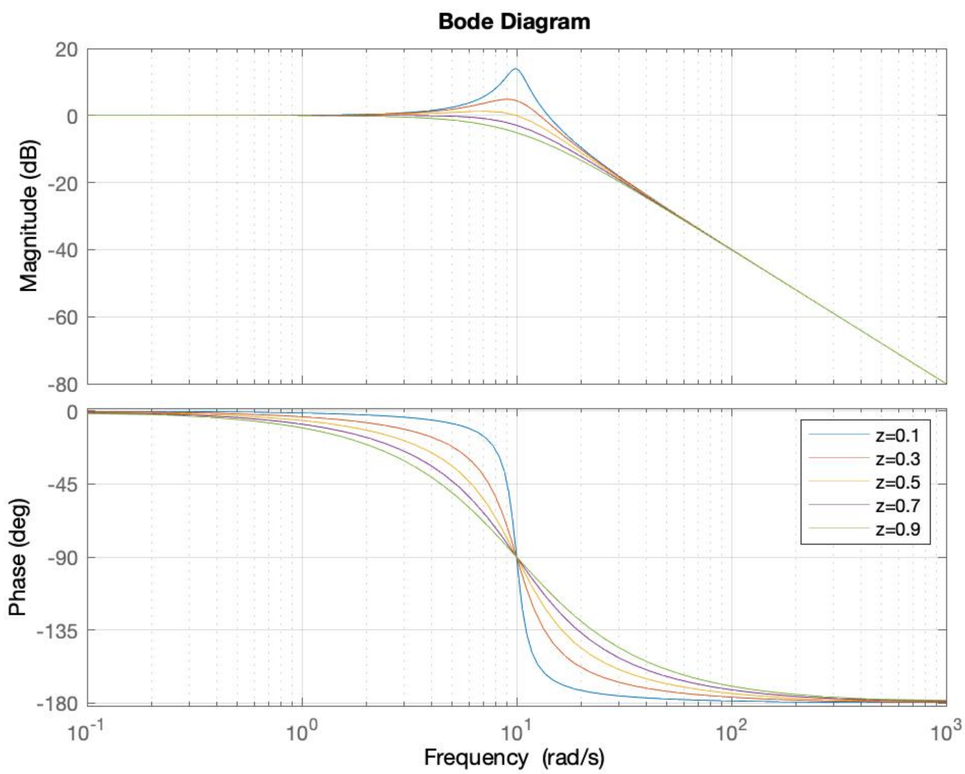
B. Assume a second-order system, here is what the bode plot looks like

$$G(s) = \frac{100}{s^2 + 4s + 100}$$



C. Key points:
   1. Phase angle ranges from 0º to 180º
   2. 2nd order systems can have a peak in them located at the damped frequency $\omega_d = \omega_n\sqrt{1 - 2\zeta^2}$
   3. On magnitude graph, horizontal at low frequencies and then -40 dB/decade over rest

D. Effect of $\zeta$ on bode plot



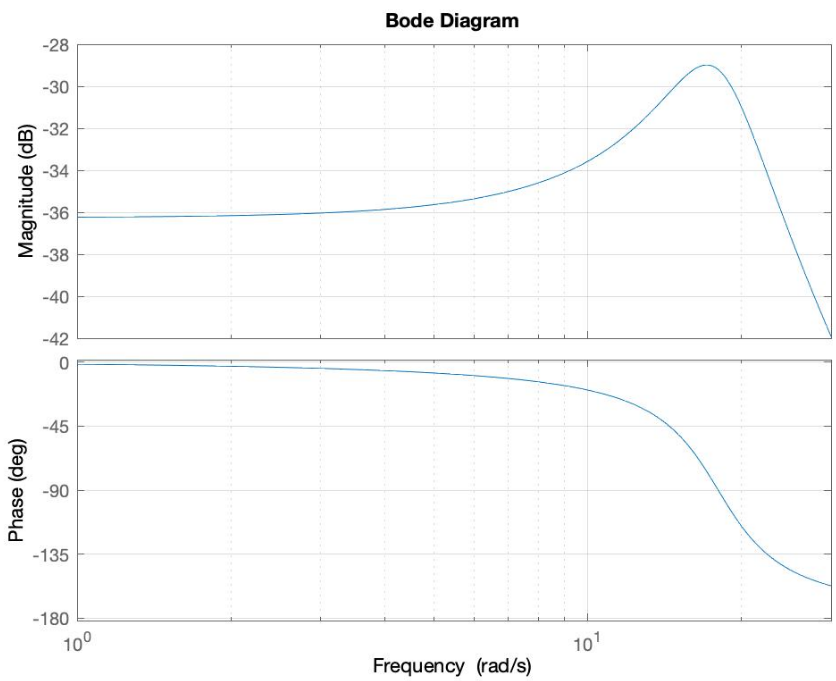

   1. Main points
      a) As $\zeta$ increases peak magnitude decreases
      b) Phase angle transition becomes sharper at lower $\zeta$
      c) Phase angle of 90º is the corner frequency $\omega_c$
      d) At $\zeta > 0.707$ there is no resonant peak

**E. Student exercise 4.** Consider the transfer function of the rotational system below. Generate a bode plot of the transfer function. Compute the frequency of the peak in the bode plot. Determine the output if the input is $T_{in}(t) = 1.5\sin(18t)$ N-m.

$$G(s) = \frac{\theta(s)}{T_{in}(s)} = \frac{1}{0.2s^2 + 1.6s + 65}$$

V. Bandwidth
   A. The bandwidth is the range of frequencies that a system will stay within 3 dB of the DC gain and up to the cutoff frequency
   B. You can visually determine this from a plot

   **C. Student exercise 5.** Estimate the bandwidth of the system response plotted below.




   D. Or you can use the matlab command `bandwidth(tf)`. Answer will be in rad

E. **Student exercise 6.** Compute the bandwidth of the system represented by the transfer function shown below

$$G(s) = \frac{5}{s^2 + 8s + 325}$$