Lesson 16
BME 444 - Control Systems

By the end of this lecture students will be able to:

- Calculate ζ and ω_n for 2nd order systems with feedback and disturbances if the controller is P, PD, or PID
- Calculate the steady-state output of a system given a step input and a P, PD, or PID controller
- Describe the effect a P, PD, or PID controller has on the system response

I. Controllers

A. We saw the effect of feedback on system response in the previous lesson.

B. In first-order systems, feedback reduces the time constant but produces a steady-state error.

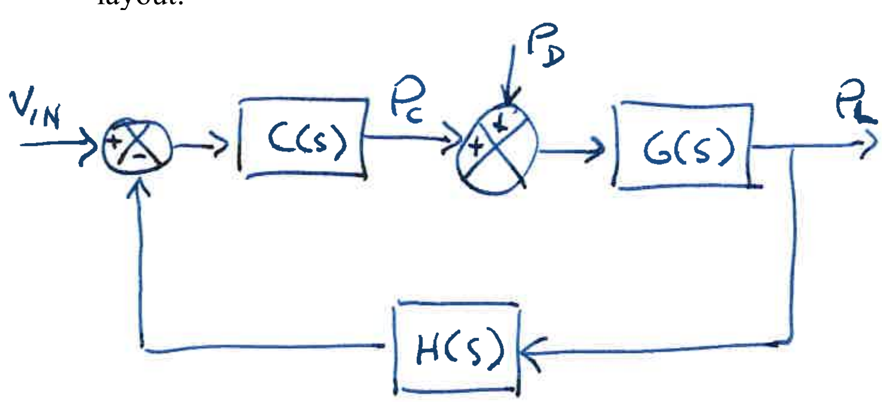
C. In second-order systems, feedback produces a steady-state error.

D. In both types of systems, feedback makes the system less susceptible to disturbances.

E. In second-order systems, controller gain can reduce ζ, making the system underdamped and potentially unstable.

F. In this lesson we focus only on second-order systems with feedback. We will look at ways to reduce the negative impacts of feedback on ζ and steady-state error (i.e., controllers).

G. All systems we consider in this lesson will have this layout:



H. Since we will only use second-order systems

$$G(s) = \frac{1}{\left(\frac{1}{\omega_n}\right)^2 s^2 + \left(\frac{2\zeta}{\omega_n}\right) s + 1}$$

I. The closed-loop feedback equation for the system above becomes

$$P_L(s) = \frac{\frac{C(s)}{1 + C(s)H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + C(s)H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right)\left(\frac{1}{1 + C(s)H(s)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1 + C(s)H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + C(s)H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right)\left(\frac{1}{1 + C(s)H(s)}\right) s + 1} P_D(s)$$

J. The difference between this system and the equations from last time is that the controller block is labeled with $C(s)$ instead of a constant and the feedback block is labeled with $H(s)$ instead of a constant.

K. This lesson will look at three types of controllers:
   1. Proportional
   2. Proportional + derivative (PD)
   3. Proportional + integral + derivative (PID)

II. Proportional controller

A. In a proportional controller, $C(s) = K_P$.

B. The equation for $P_L(s)$ becomes

$$P_L(s) = \frac{\frac{K_P}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right)\left(\frac{1}{1 + K_P H(s)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right)\left(\frac{1}{1 + K_P H(s)}\right) s + 1} P_D(s)$$

C. This equation is the same one from the previous lesson (if $H(s)$ is set equal to K_F). This type of controller is called a *proportional* controller because it multiplies the error signal by a constant, K_P.

D. Important features of this equation:
   1. At steady-state there is an error between the output and input.
   2. The natural frequency, ω_n, is modified by the presence of K_P. The new effective natural frequency of the system becomes

$$\omega_{pf} = \omega_n \sqrt{1 + K_P}$$

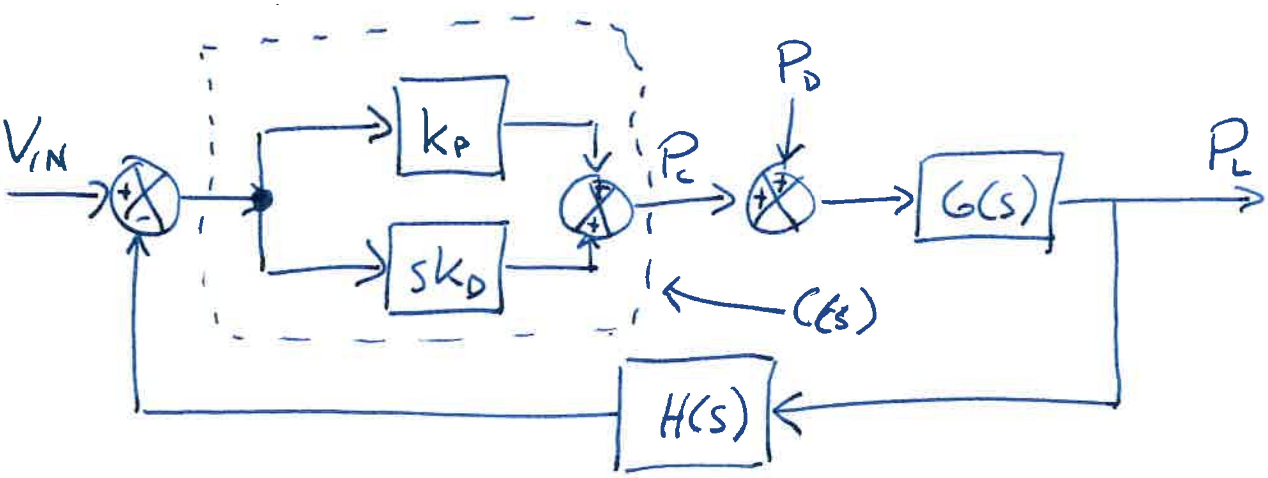
3. The damping ratio, ζ, is modified by the presence of K_P. The new effective damping ratio can be written

$$\zeta_{pf} = \frac{\zeta}{\sqrt{1 + K_P}}$$

E. **Important point.** We want K_P to be as large as possible to minimize the steady-state error. But large K_P *increases* the natural frequency and *decreases* the damping ratio. An increase in natural frequency is desirable, a decrease in damping ratio is not, so the feedback improves the system in one area while degrading it in another.

III. Proportional + derivative controller

A. We are not limited to a single controller. We can add more, like his



B. Now the controller contains a proportional controller and a derivative controller. In other words, $C(s) = K_P + K_D s$.

C. The $K_D s$ takes the derivative of the error signal and adds it to the result of the proportional controller.

D. Plug the new value for $C(s)$ into the original $P_L(s)$ equation and the equation becomes

$$P_L(s) = \frac{\frac{K_P + K_D s}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta + H(s) K_D \omega_n}{\omega_n (1 + H(s) K_P)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta + H(s) K_D \omega_n}{\omega_n (1 + H(s) K_P)}\right) s + 1} P_D(s)$$

E. Important points about this equation:
   1. The natural frequency is only controlled by K_P. We can increase K_P to minimize steady-state error without affecting the damping ratio ζ.
   2. The damping ratio is now controlled by K_P, K_D, and ω_n. We can adjust K_D without altering the other parameters (and changing ω_n) to increase ζ. Rearranging the term for ζ gives

$$\zeta_{df} = \frac{2\zeta + K_D\omega_n}{2\sqrt{1 + K_P}}$$

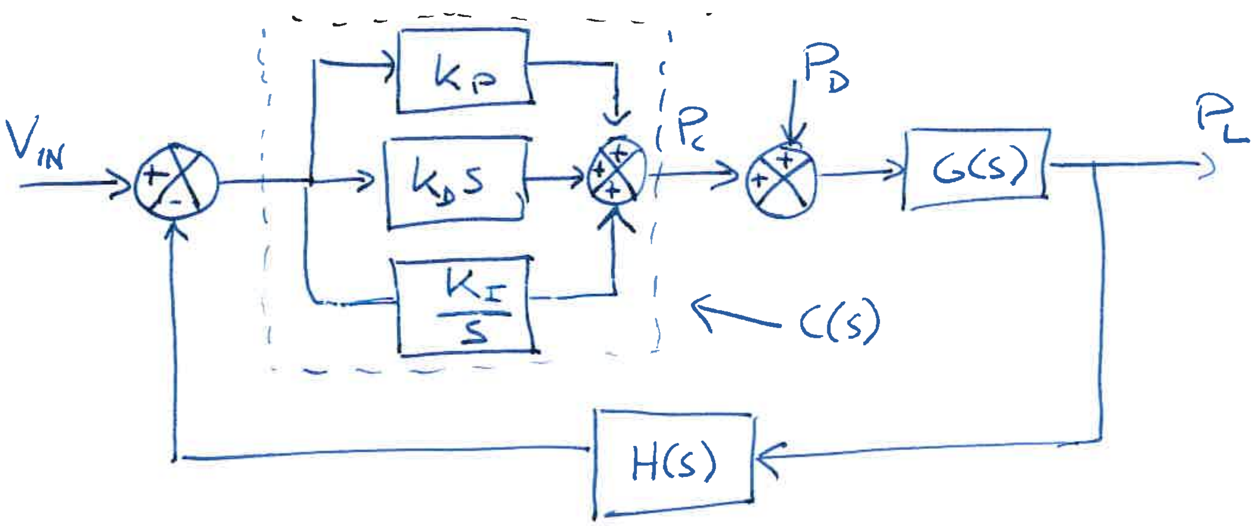
F. **Student Exercise 1:** Load the closed-loop model of your ventilator (without disturbance) from last time. Add a derivative controller to the proportional controller. Run the lung mechanics model with the following parameters. Assume a step input, R = 1 cmH20 sec/L, C = 0.01 L/cmH20, L = 0.01 cmH20 sec^2/L, $K_P = 1$, $K_D = 1$, $H = 1$, and $P_D = 0$. How does the response compare to just a proportional controller (i.e., $K_D = 0$)? What happens to the response if $K_P = 100$ and $K_D = 1$?

G. **Student Exercise 2:** Add a disturbance that is a step input of magnitude -0.5 and delay the disturbance by 0.75 seconds. Run the lung mechanics model with the following parameters. Assume a step input, R = 1 cmH20 sec/L, C = 0.01 L/cmH20, L = 0.01 cmH20 sec^2/L, $K_P = 1$, $K_D = 1$, $H = 1$. What effect does the disturbance have? Set all controller gains = 100. How does this change the response to the disturbance?

H. **Main point.** The derivative controller lets us control the steady-state error without negatively affecting damping ratio. The drawback to this type of controller is that we need an infinite value of K_P to get zero steady-state error. To tackle this problem, we need another type of controller.

IV. Proportional + integral + derivative (PID) controller

A. Now we add an integral controller to the system:



B. Now the controller has the transfer function

$$C(s) = K_P + K_D s + \frac{K_I}{s}$$

C. And plugging this value into our original equation for the ventilator gives

$$P_L(s) = \frac{\frac{K_I + K_P s + K_D s^2}{H(s)K_I}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{H(s)K_I}\right) s^3 + \left(\frac{2\zeta + H(s)K_D\omega_n}{\omega_n H(s)K_I}\right) s^2 + \left(\frac{1 + H(s)K_P}{H(s)K_I}\right) s + 1} V_{in}(s) + \frac{\frac{s}{H(s)K_I}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{H(s)K_I}\right) s^3 + \left(\frac{2\zeta + H(s)K_D\omega_n}{\omega_n H(s)K_I}\right) s^2 + \left(\frac{1 + H(s)K_P}{H(s)K_I}\right) s + 1} P_D(s)$$

D. This is a third-order system and there is not a simple analytical formula to calculate the damping ratio or natural frequency. We can make two observations about the system:

1. The natural frequency now depends on K_I instead of K_P.
2. There is no steady state error because of the integral. Cancel all terms containing s and you get $P_L = V_{in}$.

E. **Student Exercise 3:** Add a PID controller to your closed-loop system. Run the lung mechanics model with the following parameters. Assume a step input, R = 1 cmH20 sec/L, C = 0.01 L/cmH20, L = 0.01 cmH20 sec^2/L, $K_P = 1$, $K_D = 1$, $K_I = 1$, $H = 1$, and $P_D = 0$. How does the response compare to just a proportional controller (i.e., k_d = 0 and k_i = 0)?

F. **Student Exercise 4:** Add a disturbance that is a step input of magnitude -0.5 and delay the disturbance by 0.75 seconds. Run the lung mechanics model with the following parameters. Assume a step input, R = 1 cmH20 sec/L, C = 0.01 L/cmH20, L = 0.01 cmH20 sec^2/L, $K_P = 1$, $K_D = 1$, $K_I = 1$, and $H = 1$. What effect does the disturbance have? Set all controller gains = 100. How does this change the response to the disturbance?

G. **Student Exercise 5 (time permitting):** Create a Simulink file with two control systems: one with a PID and one with just P. Use a Repeating Sequence source for your disturbance with time value [0 1] and amplitude [0 2]. This type of disturbance is a ramp input and might be a leak that grows more severe over time. Use a sine wave block as your source and set the frequency to π radians/sec. Use the model parameters R = 0.01 cmH20 sec/L, C = 0.01 L/cmH20, L = 1 cmH20 sec^2/L. Run your model using a PID controller with $K_P = 1$, $K_D = 1$, $K_I = 1$. Use $K_P = 1$ for the P controller. What do the outputs of the systems look like? How does the performance of the P compare to the PID? What happens if you set $K_P = 100$ in each?