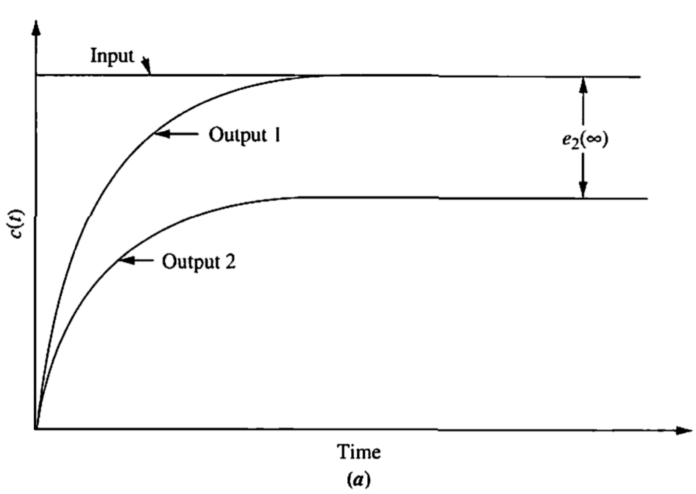
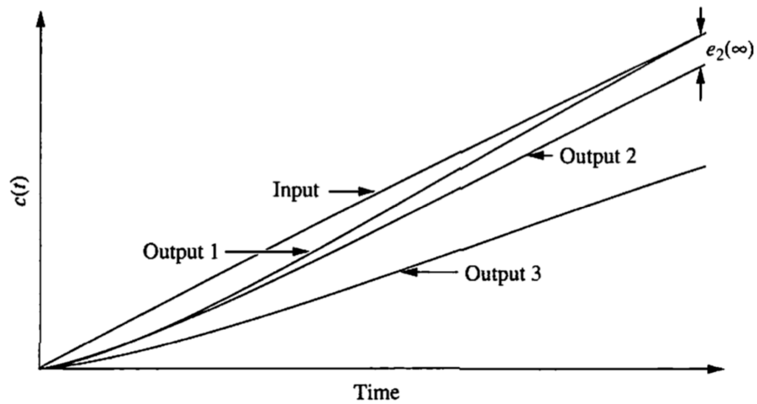
Lesson 17
BME 444 - Control Systems

By the end of this lecture students will be able to:

- Calculate steady-state error for a system with or without a disturbance for any input
- Determine system type and use this information to infer characteristics of the system
- Calculate the error constant of a system

I. Steady-state errors
   A. With the lung model, we guessed what the steady-state values would be for different systems. The shortcut was to set all $s = 0$ and then find the answer from the transfer function.
      1. *This trick only works for step inputs.*
   B. Now we practice using a method that always give the steady-state error, regardless of the type of input.
   C. The steady-state error of a system is the difference between the system input and output after a very long time, or in math lingo, as $t \rightarrow \infty$.
   D. The method shown below will work for any input, but we limit ourselves to three different types of inputs:
      1. step
      2. ramp
      3. parabola
   E. **Important.** A system must be stable for you to calculate its steady-state error. You must check the system for stability before calculating the steady-state error. (More on stability later).
   F. Steady-state error always means the difference between system input and output, but the graph of steady-state error depends on the type of input, as





(*b*)

G. Figure (a) shows the possible errors in response to a step input.
   1. No error (Output 1)
   2. Finite error (Output 2)
H. Figure (b) shows the possible errors in response to a ramp input.
   1. No error (Output 1)
   2. Finite error (Output 2)
   3. Infinite error (Output 3)
I. **Important.** You calculate the steady-state error differently for systems with a disturbance and systems without a disturbance.

II. Before you can calculate steady-state error
   A. Your system must be closed-loop with unity feedback
   B. If not, you must convert it to unity feedback before proceeding

III. Steady-state error for systems without a disturbance

A. If your unity-feedback system has already been simplified into a single block then use this formula

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[sR(s)\big(1 - T(s)\big) \right]$$

B. Otherwise, use this formula

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[sR(s) \left(\frac{1}{1 + G(s)} \right) \right]$$

C. **Example 1.** Given a unity feedback system with $G(s) = \dfrac{120(s+2)}{(s+3)(s+4)}$ the system below, find the SSE for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$.

D. **Student exercise 1.** Given a unity feedback system with $G(s) = \dfrac{100(s+2)(s+6)}{s(s+3)(s+4)}$, find the SSE for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$.

E. System type and static error constants

1. The free s in the denominator of $G(s)$ determines system type:
   a) $n = 0$ is Type 0
   b) $n = 1$ is Type 1
   c) $n = 2$ is Type 2
2. Notice that in the previous examples the results fell into three categories:
   a) 0
   b) some number
   c) ∞
3. Static error constants depend on the system type and the type of input
   a) Step

   $$e_{step}(\infty) = \frac{1}{1 + K_{pos}} \quad \text{and} \quad K_{pos} = \lim_{s \to 0} G(s)$$

   b) Ramp

   $$e_{ramp}(\infty) = \frac{1}{K_{vel}} \quad \text{and} \quad K_{vel} = \lim_{s \to 0} sG(s)$$

c) Parabola

$$e_{parabola}(\infty) = \frac{1}{K_{acc}} \text{ and } K_{acc} = \lim_{s \to 0} s^2 G(s)$$

4. In a, b, and c above, these are all for unit inputs. If the inputs are not unit, the magnitude of the input goes in the numerator of each (see ex 1 and student ex 1).
5. If you know the system type, then you can predict what the steady state errors will be

System Type, N	Unit step input	Unit ramp input	Unit parabola input
0	$\frac{1}{1+K_{pos}}$	∞	∞
1	0	$\frac{1}{K_{vel}}$	∞
2	0	0	$\frac{1}{K_{acc}}$

F. **Student exercise 2.** Consider a closed-loop system with no disturbance and unity feedback. Assume a proportional controller with gain $K_P = 0.04$. The plant transfer function is shown below. What are the steady-state errors of the system to a step, ramp, and parabola input with magnitudes of 0.01?

$$G(s) = \frac{2}{s^2 + 0.3s}$$

G. **Student exercise 3.** Consider a closed-loop system with no disturbance and unity feedback. Assume a proportional + integral controller with gain $K_P = 0.04$ and $K_I = 0.002$. The plant transfer function is shown below. What are the steady-state errors of the system to a step, ramp, and parabola input with magnitudes of 0.01?

$$G(s) = \frac{2}{s^2 + 0.3s}$$

IV. Steady-state error for systems with a disturbance

A. If you have a unity feedback system with a disturbance that looks like this

B. Then you calculate the SSE using the equation

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[\left(\frac{1}{1 + C(s)G(s)} \right) sR(s) \right] - \lim_{s \to 0} \left[\left(\frac{G(s)}{1 + C(s)G(s)} \right) sD(s) \right]$$

C. If you have a non-unity feedback system with a disturbance that looks like this

D. Then find the SSE with this equation

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\left[\left(1 - \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}\right) sR(s)\right] - \lim_{s\to 0}\left[\left(\frac{G(s)}{1 + C(s)G(s)H(s)}\right) sD(s)\right]$$

E. **Example 2.** Imagine a feedback control system with a disturbance. Find the total steady-state error due to a unit step input and a unit step disturbance. Assume

$$C(s) = \frac{1}{s+5} \text{ and } G(s) = \frac{100}{s+2} \text{ and } H(s) = 1$$

F. **Student Exercise 4.** Imagine a feedback control system with a disturbance. Find the total steady-state error if

$$r(t) = 3u(t) \text{ and } d(t) = -u(t)$$

$$C(s) = 5 \text{ and } G(s) = \frac{7}{s+2} \text{ and } H(s) = 1$$