Lesson 21
BME 444 - Control Systems

By the end of this lecture students will be able to:

- Summarize the effect of changing K_P, K_I, and K_D gains on PID controllers
- Tune PID controllers using the reaction-curve method
- Tune PID controllers using the ultimate gain method

I. PID controllers
   A. Recall that a closed-loop system with a PID controller looks like this:

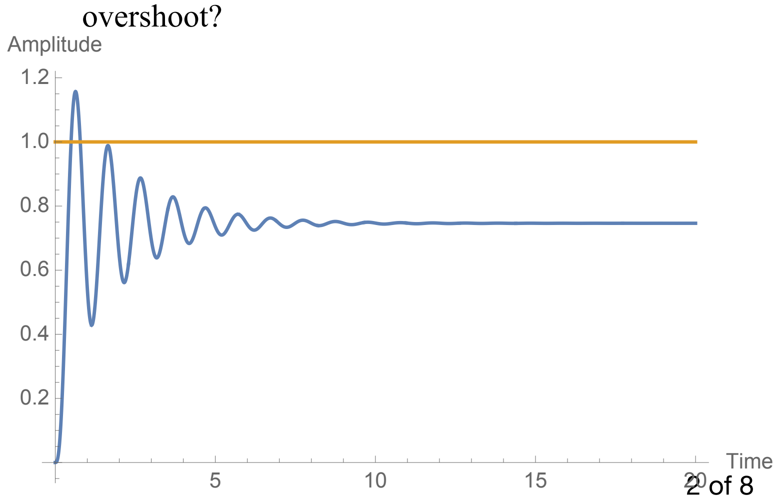
   B. Effect of each gain on the system:
      1. Proportional gain, K_P
         a) controller signal proportional to the instantaneous error
         b) increase K_P to speed up system response (reduce peak time T_P)
         c) increase K_P to reduce steady state error
         d) increasing K_P increases ω_n and decreases ζ (increases overshoot)
      2. Integral gain, K_I
         a) control signal proportional to the sum of all past errors; will be non-zero even if there is no current error

   b) increase K_I to reduce steady state error
   c) increasing K_I slows down system response (increase peak time T_P)
3. Derivative gain, K_D
   a) control signal proportional to instantaneous derivative of the error signal
   b) "anticipates" the system response because it looks at rate of change
   c) increase K_D to increase ζ (reduces overshoot and settling time)
4. Effect of increasing each gain on various parameters

	Rise Time	OS	Settling Time	SSE
K_P	Decrease	Increase	-	Decrease
K_I	Increase	Increase	Increase	Eliminate
K_D	-	Decrease	Decrease	-

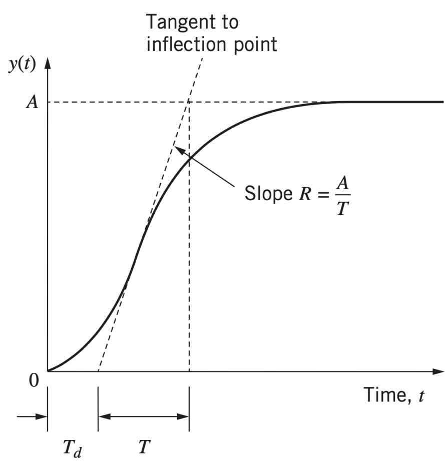
C. You may not always use K_P, K_I, and K_D depending on the system you're trying to control. For example
   1. If a system has enough damping, you might set $K_D = 0$
   2. If a system has low steady-state error, you might set $K_I = 0$

D. Student Exercise 1. A system produces the step response shown below. You know the system has a PID controller. What gain(s) would you increase/decrease to adjust the system performance to: reduce SSE, decrease rise time, and decrease overshoot?



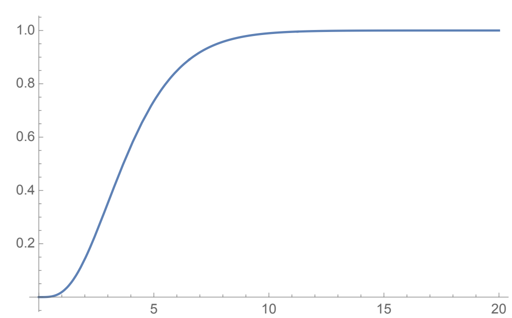
E. General guidance:
   1. Use K_P to _______________
   2. Use K_D to _______________
   3. Use K_I to _______________
F. You have three variables that you need to optimize to meet specified response time, overshoot, settling time, and SSE. How do you choose the best values? Many different strategies.
G. We cover two methods today:
   1. Ziegler-Nichols reaction-curve method
   2. Ziegler-Nichols ultimate gain method

II. Method 1: Reaction-curve method
   A. If the *plant*, $G(s)$, step response looks like an "S" curve, then you can use Ziegler-Nichols method



   B. **Step 1:** Plot the curve and extract slope and x-intercept from it (two ways): (1) calculus and (2) matlab (see F below)
      1. Calculus
      2. **Exercise 1.** Find the inflection point for the plant shown below.

Assume $G(s) = \dfrac{1}{(s+1)^4} = \dfrac{1}{s^4 + 3s^3 + 3s + 1}$



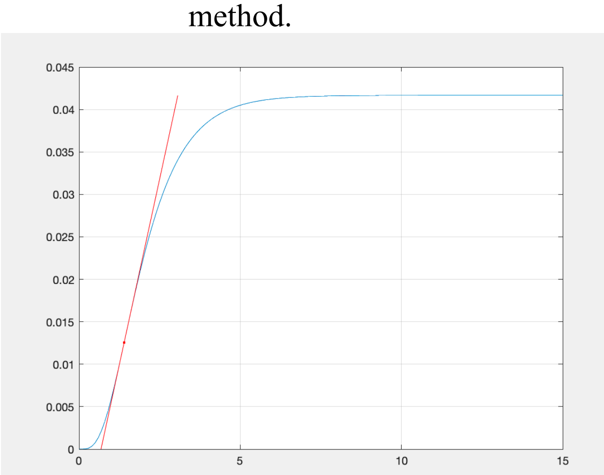
C. Step 2: Depending on the type of controller you want, use the following formulas

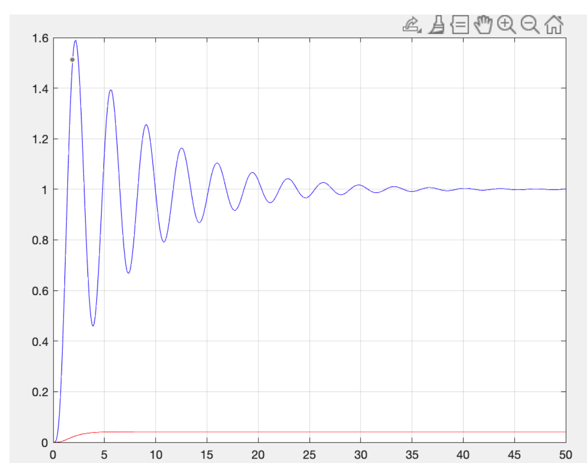
Controller Type	**Gains**
P	$K_P = \dfrac{1}{RT_d}$
PI	$K_P = \dfrac{0.9}{RT_d} \quad K_I = \dfrac{0.27}{RT_d^2}$
PID	$K_P = \dfrac{1.2}{RT_d} \quad K_I = \dfrac{0.6}{RT_d^2} \quad K_D = \dfrac{0.6}{R}$

D. If you use this method, your PID gains will have a closed-loop response that shows a 1/4 decay ratio (the transient response decays to 1/4 its peak value in one period of oscillation)

E. **NOTE:** The gain values you calculate should be treated as *starting points*. Often an engineer will adjust them to get the optimum system response

F. **Exercise 2.** Assume a plant has the transfer function $G(s) = \dfrac{1}{(s+1)(s+2)(s+3)(s+4)}$. Design a PI controller for this system using the reaction-curve method.





G. **Student Exercise 2.** You work at a company that designs and builds an infant radiant warmer, which is a medical device that keeps a newborn baby warm. In the final device, a clinician will the desired temp to 37 ºC on the control panel and an infra-red light source will gently heat the baby as needed to maintain the body temp at 37 ºC.

1. Assume that researchers in your company have developed a transfer function, $G_b(s)$, that represents the heat transfer characteristics of a human baby. Your job is to develop a PID controller to maintain the body temperature of an infant. A system diagram is shown below.

2. What gain constants does your reaction-curve method suggest?

3. Plot the step response of your closed-loop system *with controller* and the step response of the baby only (i.e., the reaction curve).
4. Which gain(s) could you change to make your system safer and more comfortable for the infant? What affect do your changes have?

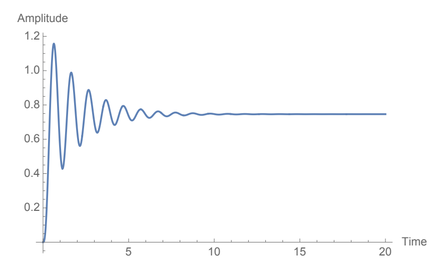
III. Method 2: Ultimate-gain method

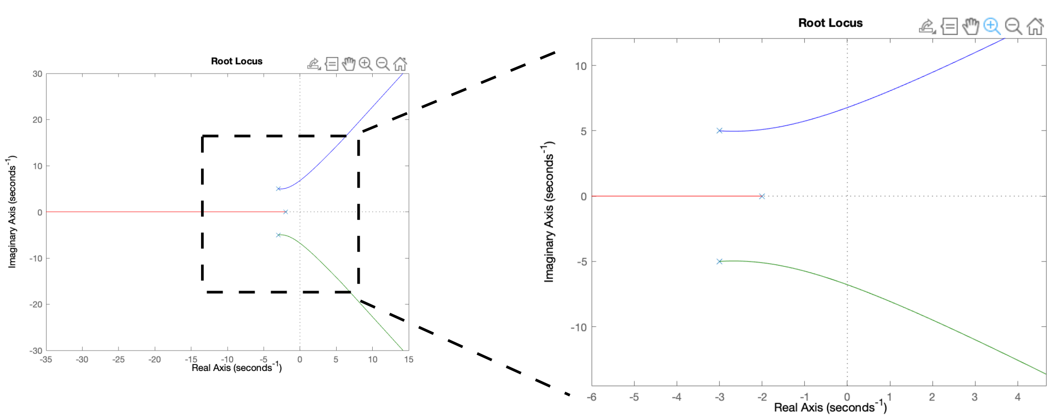
A. **Step 1:** Build your system and assume the controller is just a proportional controller with gain K_P

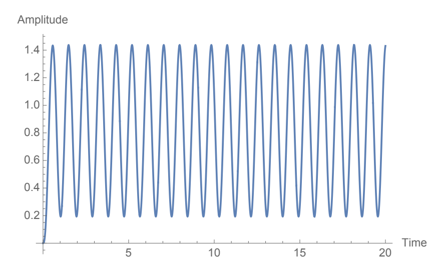
B. **Step 2:** Find the maximum K_P of the system. This gain value is known as the *ultimate gain*, K_U, of the system.

1. **NOTE:** The gain you get from the gain margin is the gain that makes the system marginally stable (i.e., when the poles are on the y-axis).
2. **Exercise 3.** Find the ultimate gain of a unity feedback system where

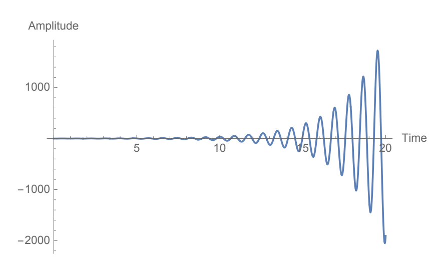
$$G(s) = \frac{1}{0.5s^3 + 4s^2 + 23s + 34}$$







C. **Step 3:** Plot the step response of the closed-loop system using $K_P = K_U$.

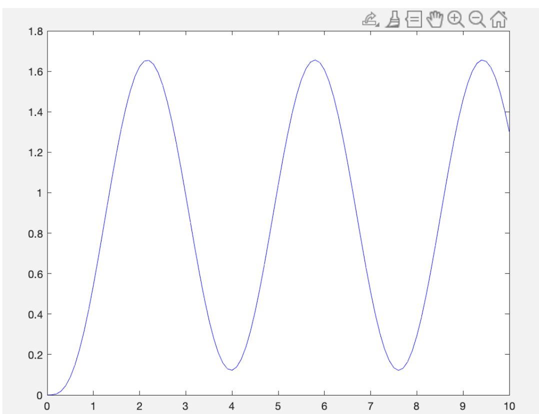


D. **Step 5:** Measure the period of the sinusoidal response and set the period equal to P_U.

E. Calculate values for K_P, K_I, and K_D using the formulas below:

Controller Type	Gains
P	$K_P = 0.5K_U$
PI	$K_P = 0.45K_U \quad K_I = \frac{0.54K_U}{P_U}$
PID	$K_P = 0.6K_U \quad K_I = \frac{1.2K_U}{P_U} \quad K_D = 0.075K_UP_U$

F. **Exercise 4.** Use the ultimate gain method to determine PID constants for the system shown in Exercise 2.



G. **Student Exercise 3.** Use the ultimate gain method to determine PID constants for the infant radiant warmer. How do they compare to the PID constants from the reaction curve method?

IV. Summing it all up
    A. You’ve seen two methods to find starting points for controller gains
    B. NOTE: not all systems will generate the “S” curve or sustained oscillations with just a P controller. If your plant does not show these responses, do not use the Ziegler-Nichols method!