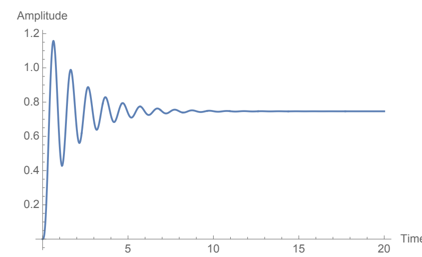
Lesson 22
BME 444 - Control Systems

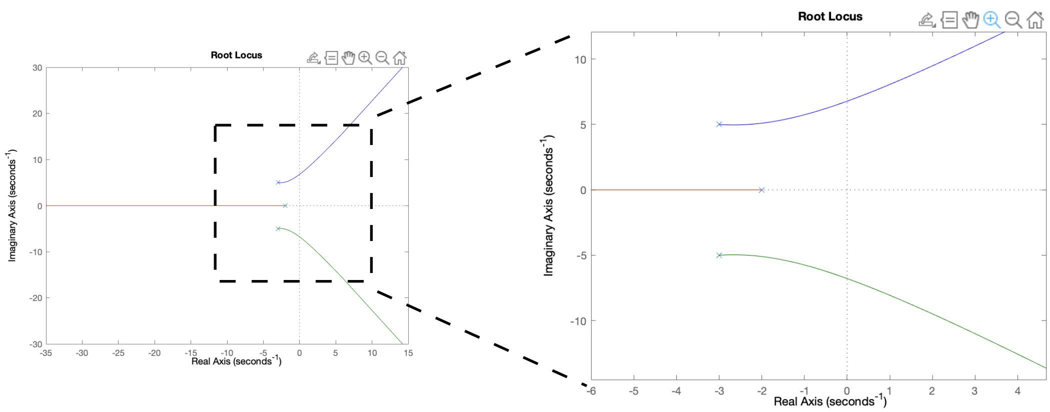
By the end of this lecture students will be able to:

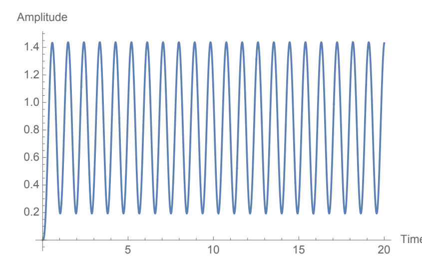
- Determine the effect of zeros on system response
- Explain how poles can move without changing system response (ζ or ω_n)
- Use root locus to design a controller
- Use frequency response to design a controller

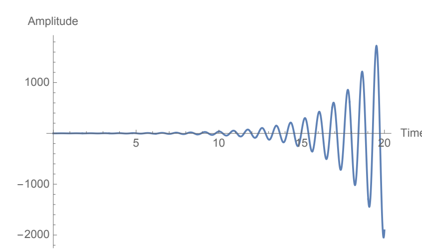


I. Re-visiting root locus plots

A. Recall that a root locus plots lets you see the effect of proportional gain on system stability



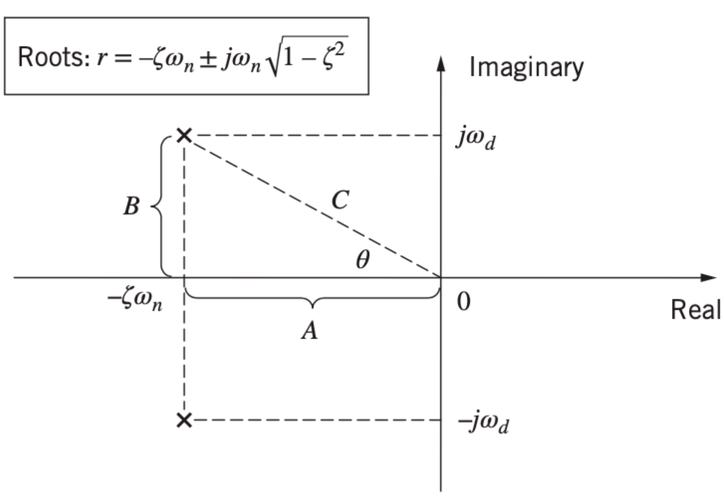




B. But, we can use root locus plots for another purpose: designing a specific system response

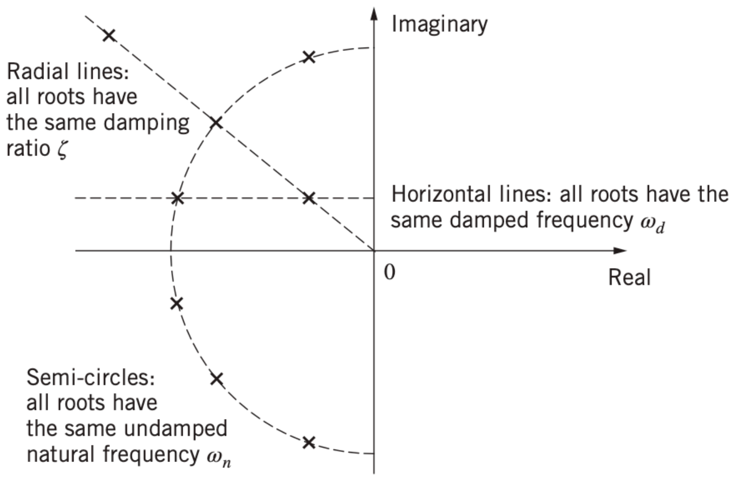
II. Pole location affects system performance

A. Pole location is determined by the roots of the transfer function denominator



B. And pole location determines system performance

C. Root locations for constant ζ, ω_n, and ω_d



D. We saw that changing the gain K_p will move the poles along the root locus plot which means we can change system performance by changing K_P

E. **Important:** But, if we adjust K_P we can only adjust system performance to places that exist on the branches (lines) of the root locus plot. *So if we want the poles in a location off the root locus plot, we have to change root location some other way (i.e., controllers/compensators)*

III. Exploring system response with root locus

A. **Exercise 1.** Create a root locus plot for a unity negative feedback system with $G(s) = \dfrac{1}{0.5s^2 + 4s + 23}$. Find a K_P that will give the system a ζ between 0.4 and 0.26. Use the sgrid command to visualize where the poles should be located to achieve the desired performance.

B. **Student Exercise 1.** Add a zero at $s = -3$ to the transfer function in Exercise 1. What effect does it have on the root locus plot? What range of system dynamics can be achieved with K_P now?

IV. Design via root locus

A. A PD controller is one way to add zeros to the system and to alter the branches of the root locus plot

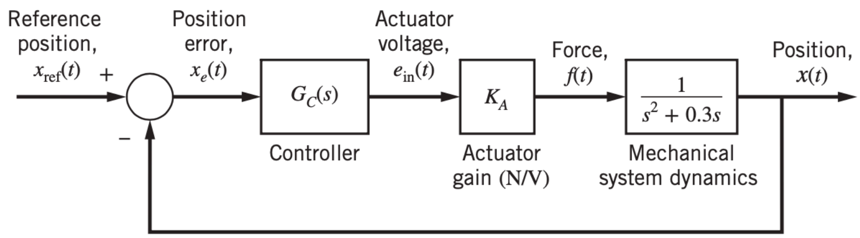
B. Recall that a PD controller takes the form

$$G_C(s) = K_P + K_D s = K(s + z_D)$$

where $K_D = K$ and $z_D = \dfrac{K_P}{K_D}$

C. Another way to think of a PD controller is that it adds a zero at $s = -z_D$ to the system, and changes the range of system performance possible.

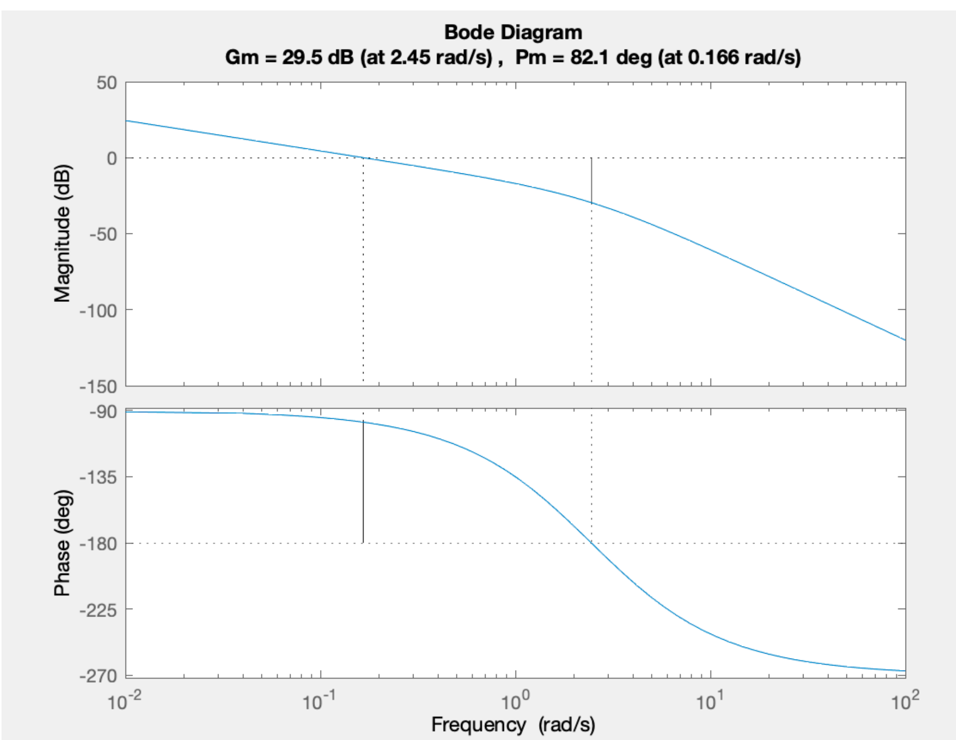
D. **Student Exercise 2.** Given the system below with $G_C(s) = K_P$ and $K_A = 2$, N/V design a P controller that will produce a fast system response ($\omega_n \approx 5.75$ rad/sec).



E. **Student Exercise 3.** Move the zero of the PD controller to $s = -3$ and $K = 1$. Can you adjust gains to achieve a fast system response ($\omega_n \approx 5.75$ rad/sec)? If so, determine values for K_P and K_D.

F. **Student Exercise 4.** What happens if you put the zero at $s = -6$?

V. Design via frequency domain
   A. Recall that gain margin and phase margin show how much gain can be increased before the system becomes unstable
   B. We ignored phase margin earlier, but we focus on it now because it provides useful information for designing controllers
   C. Example for unity feedback system with
   $$G(s) = \frac{1}{s^3 + 5s^2 + 6s}$$



D. Phase margin provides info on the damping of the system:

$$\zeta \approx \frac{\phi_{pm}}{100}$$

E. So for the system above, $\zeta \approx 0.82$.

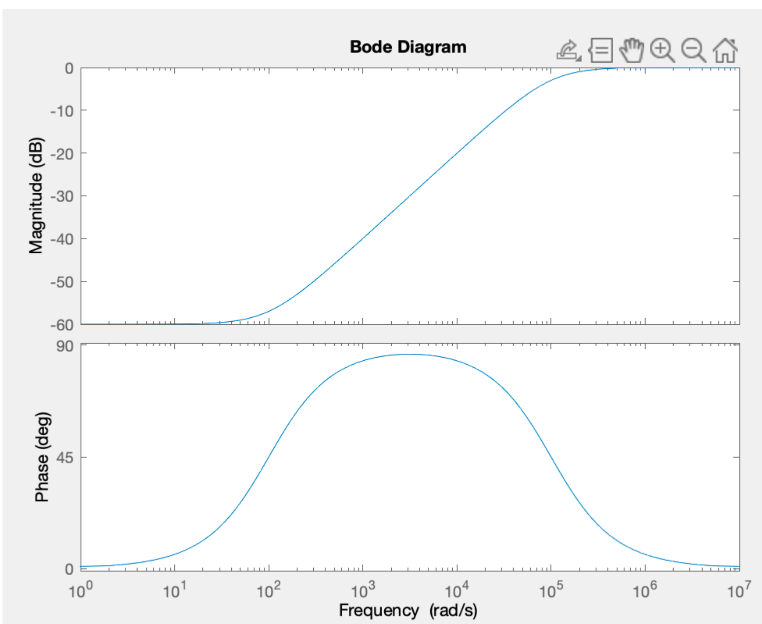
F. We can change the phase angle, and ζ, by using a lead controller

VI. Lead controllers

A. A lead controller is a controller that adds phase angle

B. The pole and zero of a lead controller control where the phase gets added

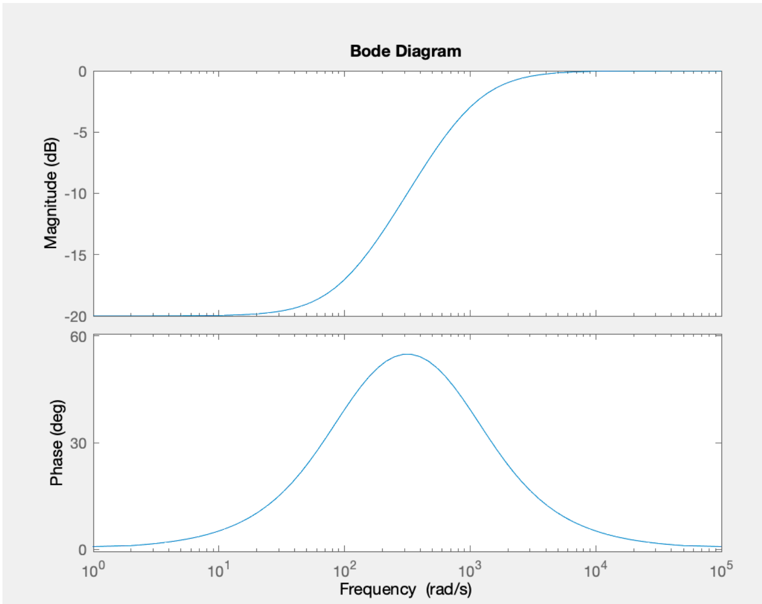
C. **Exercise 2.** Find the magnitude and phase plot of a lead controller $G_C(s) = \dfrac{s+100}{s+100000}$



D. Adjust where you want to add phase by adjusting the ω_c of the pole and zero

E. Bring phase angles close together to add less phase

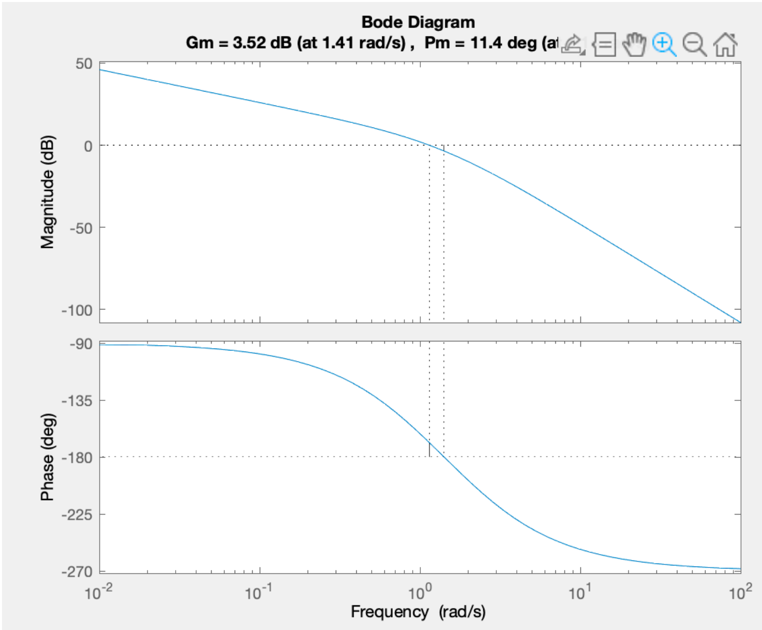
F. **Exercise 3.** Find the magnitude and phase plot of a lead controller $G_C(s) = \dfrac{s+100}{s+1000}$

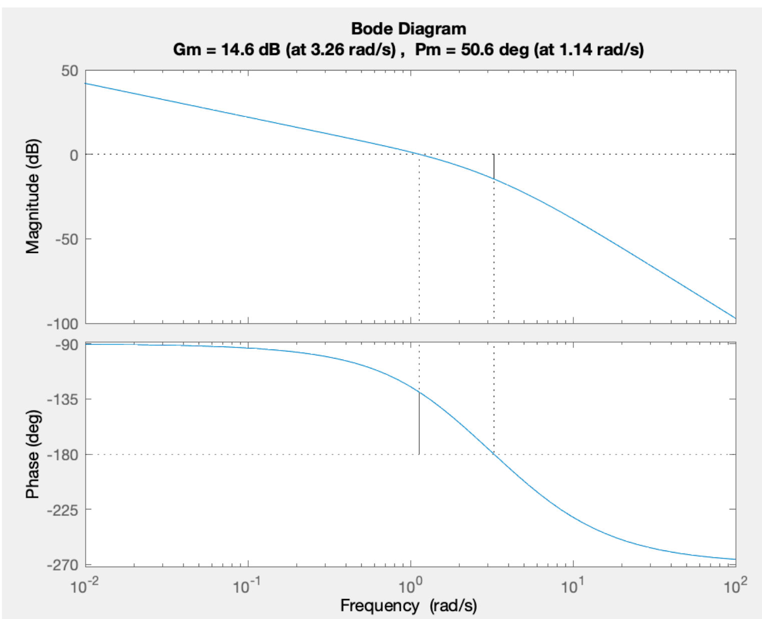


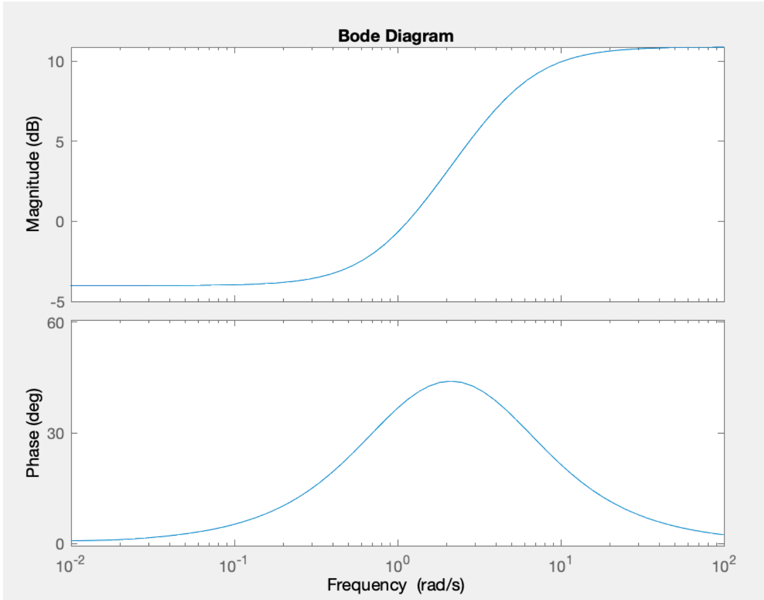
G. Minimize impact of the lead controller on the plant and other controllers by adjusting gain

H. Steps for using frequency response to design a controller
   1. **Step 1:** Determine the location and magnitude of the phase margin of the plant
   2. **Step 2:** Choose controller corner frequencies that will add to the plant phase margin and bring it to the desired ζ.
   3. **Step 3:** Note the magnitude of the controller at the phase margin frequency. Multiply a gain to bring the magnitude up to 1.

I. **Exercise 4.** Assume a unity-feedback system has a plant transfer function of $G_P(s) = \dfrac{4}{s(s+1)(s+2)}$. Design a lead controller that will make the damping ratio $\zeta \approx 0.5$.


Bode Diagram
Gm = 3.52 dB (at 1.41 rad/s) , Pm = 11.4 deg (at
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Bode Diagram
Gm = 14.6 dB (at 3.26 rad/s) , Pm = 50.6 deg (at 1.14 rad/s)
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



J. **Student Exercise 6.** Given the plant from the previous problem, design a lead controller that will make the damping ratio $\zeta \approx 0.8$.