Lesson 3
BME 444 - Control Systems

By the end of this lecture students will be able to:
- Create transfer functions of translational mechanical systems
- Create transfer functions of rotational mechanical systems
- Define linearly independent motion, degrees of freedom

1. Mathematical models of systems cont'd

Translational Mechanical Systems

Mechanical systems are analogous to electrical systems in that there are three types of components: springs, dashpots, and masses.

When using mesh analysis with circuits, the number of equations equals the number of loop currents. In mechanical systems, the number of equations equals the number of linearly independent motions.

Important point. A *linearly independent motion* is a point in the mechanical system that can move if everything else is held still; *this is usually a mass*. The number of linearly independent motions is equal to the *degrees of freedom* of the system.

Example 1: Single mass (analogous to a single loop RLC circuit)

Given:

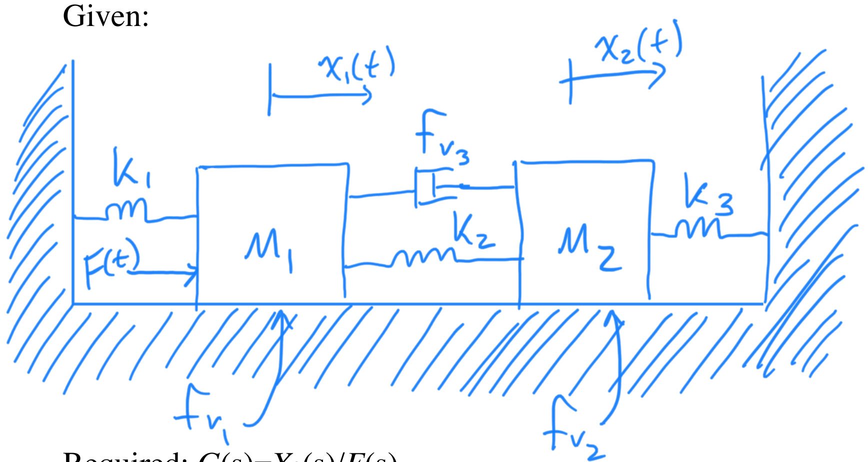
Required: $G(s)=X(s)/F(s)$

Steps for solving translational mechanical problems:

1. Hold all points of motion still except for current mass
2. Sum forces acting on current mass. Write $f_v s$ for dashpot, K for spring, and Ms^2 for mass.
3. Hold current mass still and move all others. Subtract forces acting on mass.
4. Solve the resulting equations of motion.

Example 2:

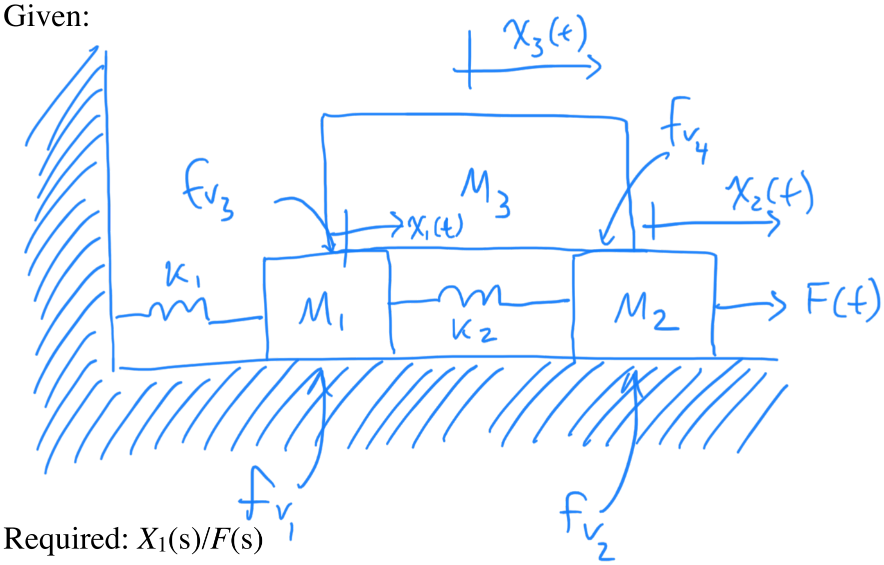
Given:



Required: $G(s)=X_2(s)/F(s)$

Student Example 1:

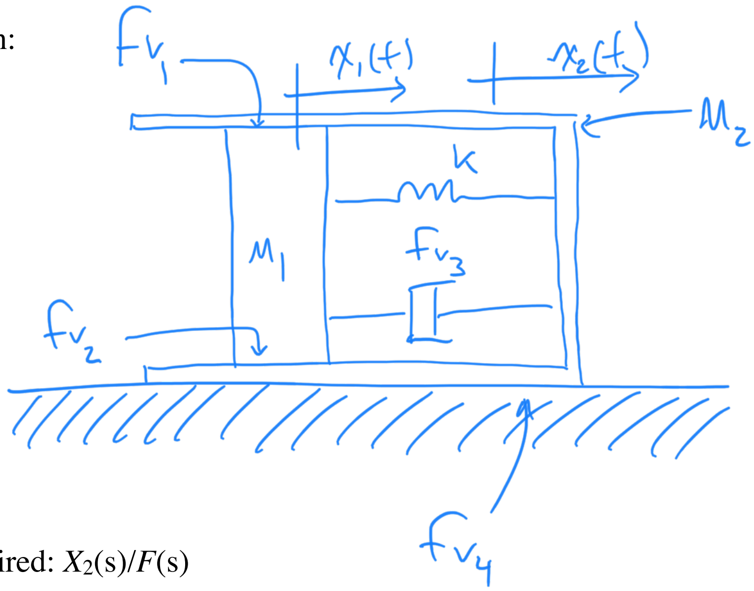
Given:



Required: $X_1(s)/F(s)$

Student Example 2:

Given:



Required: $X_2(s)/F(s)$

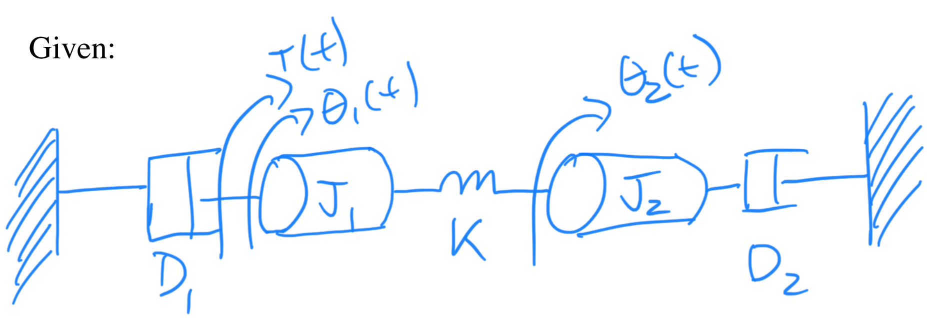
Rotational Mechanical Systems

Mechanical systems can also be described by rotational parts. A good example is the inner workings of a clock based on gears. The same overall themes that were used with translational systems apply to rotational systems as well.

Use the same steps for rotational systems as we used with translational systems to solve these problems.

Example 3:

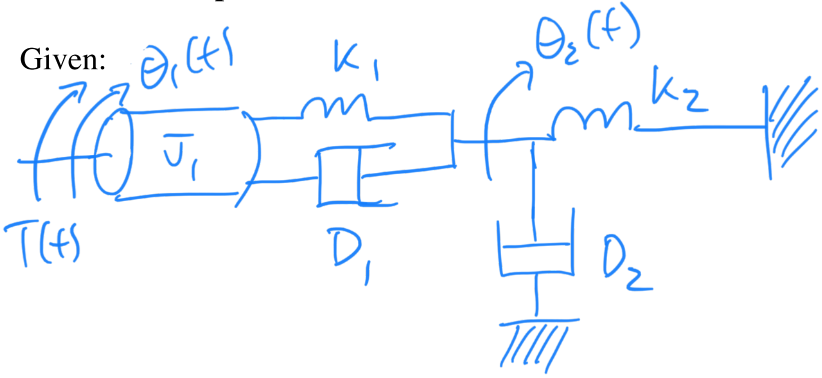
Given:



Required: $\theta_2(s)/T(s)$

Student Example 3:

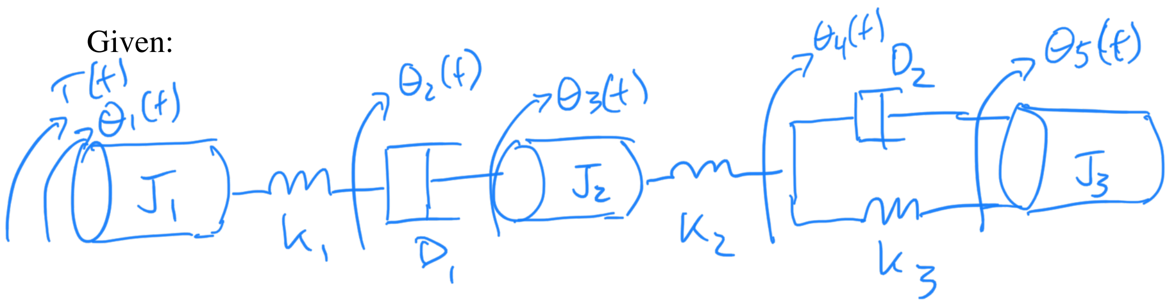
Given:



Required: $\theta_2(s)/T(s)$

Student Example 4:

Given:



Required: $\theta_2(s)/T(s)$