Lesson 7
BME 444 - Control Systems

By the end of this lecture students will be able to:
- Convert a transfer function to state space and vice versa
- Convert a transfer function to a DE and vice versa

I. Where we are

A. The state-space representation (SSR) is a good place to stop; this represents the model in a form in which an input and output can be simulated anywhere
B. However, transfer functions provide great insight into system behavior, so it would be useful if we can convert from SSR to transfer functions

II. Converting from SSR to TF
A. There are two ways:
1. linear algebra
2. MATLAB function ss2tf()

B. Linear Algebra

1. Remember that the SSR looks like this:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

2. If we take the LaPlace transform of these two equations and combine them, we get the following formula

$$Y(s) = (C(sI - A)^{-1}B + D)U(s)$$

- In this equation, I is the identity matrix
- It can vary in size & has 1's in the diagonal
- A 3×3 identity matrix looks like

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and so on

3. If we re-arrange the above equation a little, we get

$$\frac{Y(s)}{U(s)} = (C(sI - A)^{-1}B + D)$$

4. Which is a transfer function. So, to convert SSR to a transfer function, use the above formula.
5. **Example 1.** The SSR for Example 1 from last class is written below. Convert it to a transfer function using the linear algebra method.

$$\dot{x} = \begin{bmatrix} \dot{v_{C1}} \\ \dot{v_{C2}} \\ \dot{i_L} \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC_1} & \frac{-1}{RC_1} & \frac{1}{C_1} \\ \frac{-1}{RC_2} & \frac{-1}{RC_2} & 0 \\ \frac{-1}{L} & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{RC_2} \\ \frac{1}{L} \end{bmatrix} v(t) \qquad y = [0 \quad 1 \quad 0] \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix}$$

```
clear all;
syms s r l c1 c2
A=[-1/(r*c1) -1/(r*c1) 1/c1; -1/(r*c2) -1/(r*c2) 0;
-1/l 0 0]
B=[1/(r*c1); 1/(r*c2); 1/l]
C=[0 1 0]
I=eye(3);
D=0;
ans = C*inv(s*I-A)*B+D
simplify(ans)
```

6. **Student Example 1.** The SSR for Student Example 1 from last class is written below. Convert it to a transfer function using the linear algebra method.

$$\dot{x} = \begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v(t) \qquad y = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

C. Matlab function

1. The Control Systems toolbox in MATLAB contains the function ss2tf() that will perform the conversion.
2. **Example 2.** Repeat Example 1 using ss2tf().

3. **Student Example 2.** Repeat Student example 1 using ss2tf(). Assume R = L = 1 and C = 1e-6.

III. Converting from TF to SSR

A. What if we're given a transfer function and want to calculate the state-space representation?

B. We can do this, but remember: **the state-space representation (SSR) you calculate from a transfer function will not be the same SSR you get if you start with the physical system and derive it yourself**

1. That's okay because both are still "correct"
2. They just won't be the same state-space representations
3. You will not know what the state variables represent when you go from TF to SS—they will still be valid state variables, but they will be abstract.

C. There are two ways to do this:

1. The canonical method
2. MATLAB function tf2ss()

D. **Example 3.** Given a transfer function for a system show below, convert to SSR using the canonical method.

$$G(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

E. **Student example 3.** Given a transfer function for a system show below, convert to SSR using the canonical method.

$$G(s) = \frac{2s + 1}{s^2 + 7s + 9}$$

1. **Example 4.** Given the transfer function below, use tf2ss() in MATLAB to convert to SSR.

$$G(s) = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

2. **Student Example 4.** Given the transfer function below, use tf2ss() in MATLAB to convert to SSR.

$$G(s) = \frac{2s + 1}{s^2 + 7s + 9}$$

IV. Converting the TF to a DE and vice versa

A. The transfer function is just a LaPlace transform of a differential equation written in a non-standard way

B. **Example 5.** Given the transfer function below, convert it to a differential equation.

$$G(s) = \frac{7s + 4}{2s^2 + 10s + 28}$$

C. **Student example 5.** Convert the differential equation below to a transfer function.

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 10y = 4\frac{du}{dt}$$