

Lesson 8

BME 444 - Control Systems

By the end of this lecture students will be able to:

- Use MATLAB commands or Simulink to simulate the response of a system to inputs
- Simulate systems using transfer functions, state-space representations, or integrator-blocks

I. Overview

- A. We've practiced representing systems as equations
- B. Now it's time to learn how to give the systems inputs and visualize the outputs
- C. We will practice two methods of simulating systems
 1. Matlab commands
 2. Simulink
- D. Within Simulink, we will explore three different methods of building models
 1. Transfer functions (TF)
 2. State-space representation (SSR)
 3. Integrator blocks (IB)
- E. Each approach has advantages and disadvantages, as summarized in the table below

	TF	SSR	IB
Allows initial conditions?	N	Y	Y
Linear/Non-linear	Linear	Linear	Linear or Non-linear
Insight	Y	N	N

II. Command-line simulations

- A. We defined four input types last time. In MATLAB they are:
 1. `u = eq(t,0); % impulse input`
 2. `u = heaviside(t); % unit step`
 3. `u = t; % ramp`
 4. `u = sin(2*t); % sin(wt)`
- B. Shortcuts: you can use `step()` and `impulse()` but these assume all initial conditions in the system are = 0

- C. The function `tf()` creates a transfer function object in MATLAB
- D. The function `lsim()` can
 1. Simulate transfer functions or state space
 2. Use any input function
 3. Handle initial conditions if state-space is used
- E. **Example 1.** Plot the unit impulse response and step response of the DE shown below

$$\ddot{y} + 3\dot{y} + 12y = 0.8u$$

- F. **Student example 1.** Given the RL circuit shown below, use MATLAB to determine the current in the loop and the voltage $v_0(t)$. Assume $v_{in}(t) = u(t)$, zero initial conditions, and $R = 1.6 \Omega$ and $L = 0.1$ H.

G. **Example 2.** Implement a state-space representation with `lsim()`.

```
A = [0 1; -12 -3];  
B = [0; 0.8];  
C = [1 0];  
D = 0;  
sys = ss(A,B,C,D);
```

Then everything else the same as before

H. Define initial conditions in state-space like this:

```
[y,t] = lsim(sys, u, t, x0);
```

Where x_0 is the initial state vector (the starting values of each state variable)

III. Simulink - Transfer functions

A. Simulink is a graphical way to simulate systems

B. Behind the scenes, it is numerically solving the differential equations; in other words, it is computing actual values from the equations themselves

C. **Example 3.** Implement the RL circuit example in Simulink. Plots outputs of the current and $v_0(t)$.

Under Modeling>Model Settings choose ode4 and fixed time step of 10^{-3} . For convenience, the transfer function is

$$\frac{I(s)}{V_{in}(s)} = \frac{1}{Ls + R}$$

- D. Write output to workspace if you want to analyze it in MATLAB
- E. Use Scope for output if you want to see the graph
- F. **Student example 2.** Use Simulink to model the following differential equation of a mechanical system as a transfer function. Assume a step input of magnitude 12 N is applied at $t = 0.02$ sec. Run the simulation to 0.1 sec.

$$0.04\ddot{y} + 16\dot{y} + 7000y = u$$

IV. Simulink - State Space

- A. State space works essentially the same way as with transfer functions
- B. Just choose the state space box instead of the transfer function box
- C. Be sure that the dimensions on your **D** vector are correct (down x across)
 1. **A** matrix: $n \times n$
 2. **B** matrix: $n \times r$
 3. **C** matrix: $m \times n$
 4. **D** matrix: $m \times r$
- D. Define the initial conditions for each state space variable as appropriate
- E. **Example 4.** Implement the following state space representation in Simulink. Assume the input is a step input with magnitude 12 N applied at $t = 0$ sec. Run the simulation to $t = 0.1$ sec.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -175,000 & -400 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u(t) \quad y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

V. Simulink - Integrator block method

- A. This method is the easiest to implement because you are basically re-creating the equation using the blocks
- B. **Example 5.** Implement the following DE in Simulink using the integrator block method. Assume a step input applied at $t = 0$ sec with a

magnitude of 12 N. Run the simulation to $t = 0.1$ sec.

$$0.04\ddot{y} + 16\dot{y} + 7000y = u$$

C. Student example 3. Use Simulink to simulate a non-linear model of the hydraulic tank system shown below. Assume the equation is

$$C\dot{P} = Q_{in} - K_T\sqrt{P - P_{atm}}$$

The input is Q_{in} and the output is P , tank base pressure

Assume the following:

$$C = 0.0002 \text{ m}^3/\text{Pa}$$

$$K_T = 4 \times 10^{-4} \text{ m}^{3.5}/\text{kg}^{0.5}$$

$$Q_{in} = 0.052 \text{ m}^3/\text{s}$$

$$P_{atm} = 1.0133 \times 10^5 \text{ N/m}^2$$

$$P_0 = 1.15 \times 10^5 \text{ N/m}^2$$

Note the P_0 , which is the initial pressure in the tank.

Run the simulation to $t = 1400$ sec

