

Lesson 16

BME 444 - Biomedical Controls

Contents

1	Learning Objectives	1
2	Introduction	2
3	Second-Order Model	2
	3.1 Open-Loop Second-Order System . .	2
	3.2 Closed-Loop Second-Order System .	3
	3.3 Student Exercise 1	3
	3.4 Student Exercise 2	4
	3.5 Summary of Second-Order Systems .	4
4	Controllers	4
	4.1 System Layout	5
5	Proportional Controller	6
6	Proportional + Derivative Controller	7
	6.1 Student Exercise 3	8
	6.2 Student Exercise 4	8

1 Learning Objectives

By the end of this lesson students will be able to:

- Explain the difference between second-order transient system responses with and without feedback
- Explain the effect controller gain has on system output for second-order systems
- Calculate the output and/or transfer function of a system with feedback and a disturbance
- Calculate the steady state value for second-order systems with feedback and a disturbance, given a step input
- Calculate ζ and ω_n for second-order systems with feedback and disturbances if the controller is P or PD

- Calculate the steady-state output of a system given a step input and a P or PD controller
- Describe the effect a P or PD controller has on the system response

2 Introduction

We continue with our exercise to design a mechanical ventilator. We create a second-order model of the lung and investigate how the system response changes with this more accurate model. Then we start to investigate the effect of different controllers on system response.

3 Second-Order Model

To make the lung model more accurate, we need to also model the inertia of moving air. An inductor models the inertia of mass.

First, derive $G(s) = \frac{P_L(s)}{P_{in}(s)}$

Now that we have the controlled system transfer function, we can simulate the open-loop and closed-loop system response to various inputs.

3.1 Open-Loop Second-Order System

Using the open-loop system diagram from Lesson 15 as a guide, we write an equation for the output of the system. When done, we replace $G(s)$ with the second-order system.

3.2 Closed-Loop Second-Order System

Next, using the closed-loop system diagram from exercise 1 as a guide, we write an equation for the output of the system. When done, we replace $G(s)$ with the second-order system.

3.3 Student Exercise 1

Compare the responses of the open- and closed-loop systems in Simulink. Assume a step input of magnitude 1, $R = 1 \text{ cmH}_2\text{O sec/L}$, $C = 0.1 \text{ L/cmH}_2\text{O}$, $L = 0.01 \text{ cmH}_2\text{O sec}^2/\text{L}$, $K_C = 1$, $K_F = 1$, and $d = 0$. Write the steady-state value for the open-loop and closed-loop systems below. Run the simulation again with $K_C = 100$.

Results

3.4 Student Exercise 2

In the model you used in the previous exercise, add a disturbance that is a step input of magnitude -0.5 and delay the disturbance by 0.75 seconds. Set $K_C = 1$ and run both models. Record the steady-state value. Set $K_C = 100$ and run again.

Results

3.5 Summary of Second-Order Systems

1. Feedback introduces a steady-state error
2. An increase in gain increases the output of an open-loop system but reduces steady-state error in a closed-loop system; *we cannot eliminate steady-state error completely*
3. Feedback and high gain reduce the impact of disturbances, but *may alter type of system response*

Be sure to save your models. We will continue using them.

4 Controllers

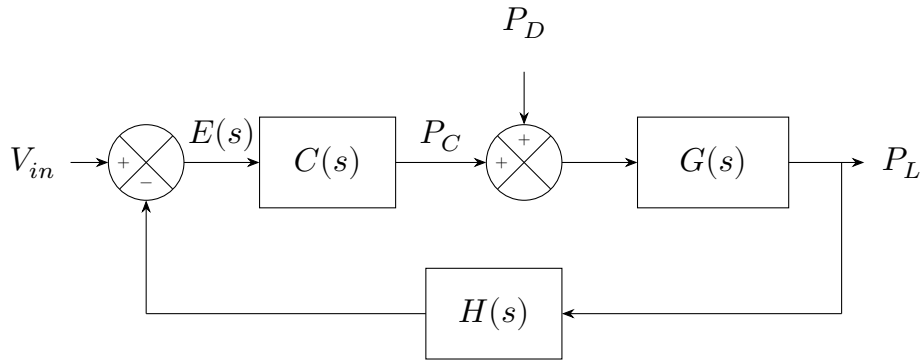
We have seen the effect of feedback on system response.

- In first- and second-order systems, feedback reduces the time constant but produces a steady-state error.
- In both types of systems, feedback makes the system less susceptible to disturbances.
- In second-order systems, increasing controller gain reduces ζ , which can make the system underdamped.

For the rest of this lesson, we focus only on second-order systems with feedback. We will look at ways to reduce the negative impacts of feedback on ζ and steady-state error (i.e., controllers).

4.1 System Layout

When we talk about controllers, all systems will have this layout:



Since we will only use second-order systems:

$$G(s) = \frac{1}{\left(\frac{1}{\omega_n}\right)^2 s^2 + \left(\frac{2\zeta}{\omega_n}\right) s + 1}$$

The closed-loop feedback equation for the system above becomes

$$P_L(s) = \frac{\frac{C(s)}{1+C(s)H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1+C(s)H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right) \left(\frac{1}{1+C(s)H(s)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1+C(s)H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1+C(s)H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right) \left(\frac{1}{1+C(s)H(s)}\right) s + 1} P_D(s)$$

The difference between this system and the equations for a first-order system is that the controller block is labeled with $C(s)$ instead of a constant and the feedback block is labeled with $H(s)$ instead of a constant.

We will look at three types of controllers that can be plugged into the $C(s)$ block:

1. Proportional

2. Proportional + derivative (PD)
3. Proportional + integral + derivative (PID)

5 Proportional Controller

In a proportional controller, $C(s) = K_P$.

The equation for $P_L(s)$ becomes

$$P_L(s) = \frac{\frac{K_P}{1+K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1+K_P H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right) \left(\frac{1}{1+K_P H(s)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1+K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1+K_P H(s)}\right) s^2 + \left(\frac{2\zeta}{\omega_n}\right) \left(\frac{1}{1+K_P H(s)}\right) s + 1} P_D(s)$$

This equation is the same one from earlier in this lesson (if $H(s)$ is set equal to K_F). This type of controller is called a *proportional* controller because it multiplies the error signal by a constant, K_P .

Important features of this equation:

1. At steady-state there is an error between the output and input.
2. The natural frequency, ω_n , is modified by the presence of K_P . The new effective natural frequency of the system becomes

$$\omega_{pf} = \omega_n \sqrt{1 + K_P}$$

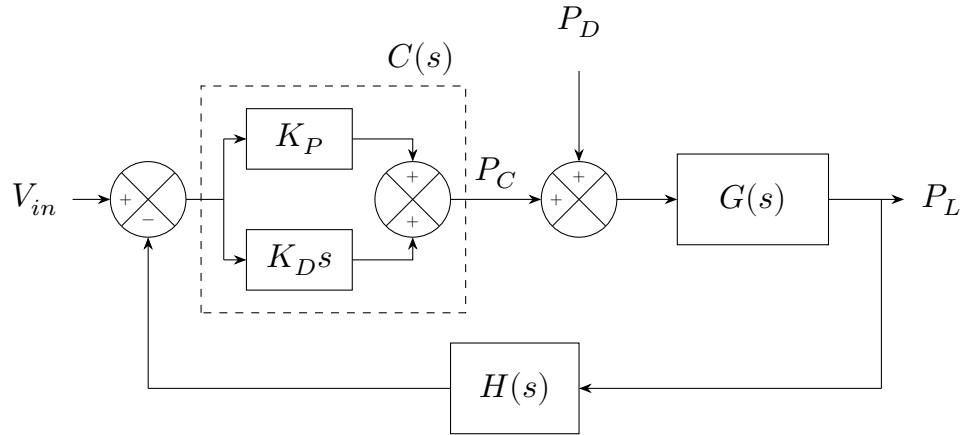
3. The damping ratio, ζ , is modified by the presence of K_P . The new effective damping ratio can be written

$$\zeta_{pf} = \frac{\zeta}{\sqrt{1 + K_P}}$$

Important point. We want K_P to be as large as possible to minimize the steady-state error. But large K_P *increases* the natural frequency and *decreases* the damping ratio. An increase in natural frequency is desirable, a decrease in damping ratio is not, so the feedback improves the system in one area while degrading it in another.

6 Proportional + Derivative Controller

We are not limited to a single controller. We can add more, like this:



Now the controller contains a proportional controller and a derivative controller. In other words, $C(s) = K_P + K_D s$.

The $K_D s$ takes the derivative of the error signal and adds it to the result of the proportional controller.

Plug the new value for $C(s)$ into the original $P_L(s)$ equation and the equation becomes

$$P_L(s) = \frac{\frac{K_P + K_D s}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta + H(s)K_D \omega_n}{\omega_n(1 + H(s)K_P)}\right) s + 1} V_{in}(s) + \frac{\frac{1}{1 + K_P H(s)}}{\left(\frac{1}{\omega_n}\right)^2 \left(\frac{1}{1 + K_P H(s)}\right) s^2 + \left(\frac{2\zeta + H(s)K_D \omega_n}{\omega_n(1 + H(s)K_P)}\right) s + 1} P_D(s)$$

Important points about this equation:

1. The natural frequency is only controlled by K_P . We can increase K_P to minimize steady-state error without affecting the damping ratio ζ .
2. The damping ratio is now controlled by K_P , K_D , and ω_n . We can adjust K_D without altering the other parameters (and changing ω_n) to increase ζ .
Rearranging the term for ζ gives

$$\zeta_{df} = \frac{2\zeta + K_D \omega_n}{2\sqrt{1 + K_P}}$$

6.1 Student Exercise 3

Load the closed-loop model of your ventilator (without disturbance) from last time. Add a derivative controller to the proportional controller. Run the lung mechanics model with the following parameters. Assume a step input, $R = 1$ cmH₂O sec/L, $C = 0.01$ L/cmH₂O, $L = 0.01$ cmH₂O sec²/L, $K_P = 1$, $K_D = 1$, $H = 1$, and $P_D = 0$. How does the response compare to just a proportional controller (i.e., $K_D = 0$)? What happens to the response if $K_P = 100$ and $K_D = 1$?

Solution

6.2 Student Exercise 4

Using the model you developed at the end of the last class, add a disturbance that is a step input of magnitude -0.5 and delay the disturbance by 0.75 seconds. Run the lung mechanics model with the following parameters. Assume a step input, $R = 1$ cmH₂O sec/L, $C = 0.01$ L/cmH₂O, $L = 0.01$ cmH₂O sec²/L, $K_P = 1$, $K_D = 1$, $H = 1$. What effect does the disturbance have? Set all controller gains = 100. How does this change the response to the disturbance?

Solution

Main point. The derivative controller lets us control the steady-state error without negatively affecting damping ratio. The drawback to this type of controller is that we need an infinite value of K_P to get zero steady-state error. To tackle this problem, we need another type of controller.