BME 444 — HW 2 Key

Problem 1

The answer should be a state-space representation of a circuit. Assume the output is $v_o(t)$. Note that $v_o(t) = v_c(t)$ based on the problem given.

$$\dot{x} = \begin{bmatrix} \dot{i}_2 \\ \dot{i}_4 \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} v(t)$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ v_c \end{bmatrix}$$

Problem 2

The answer should be a state-space representation of a system. Assume the output is $x_3(t)$.

$$\dot{\mathbf{Z}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1.5 & 1 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & -3 \end{bmatrix} \mathbf{Z} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f(t)$$

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{Z}$$

Problem 3

The two answers should be state-space representations of traditional transfer functions.

(a)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -13 & -5 & -1 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$c = \begin{bmatrix} 10 & 8 & 0 & 0 \end{bmatrix} \mathbf{x}$$

(b)

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -8 & -13 & -9 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$c = \begin{bmatrix} 6 & 7 & 12 & 2 & 1 \end{bmatrix} \mathbf{x}$$

Problem 4

The three answers to this problem should be transfer functions for given state-space representations.

(a)

$$\frac{10}{s^3 + 5s^2 + 2s + 3}$$

(b)

$$\frac{49s^2 - 373s + 680}{s^3 - 3s^2 - 27s + 103}$$

(c)

$$\frac{23s^2 - 48s - 7}{s^3 + 3s^2 + 19s - 133}$$

Problem 5

The answer should be the state-space representation of a system of five interdependent linear first-order differential equations.

$$\begin{bmatrix} \dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} a_{00} & 0 & a_{02} & 0 & 0 \\ a_{10} & a_{11} & a_{12} & 0 & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{32} & a_{33} & 0 \\ 0 & 0 & a_{42} & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} d_0$$