BME 444
Glucose Homeostasis 1

1. Mass balance equations?

$$\frac{dGLU}{dt} = \dot{m}_{inGLU} - \gamma INS - \delta GLU \qquad \frac{dINS}{dt} = \dot{m}_{inINS} - \alpha INS + \beta GLU$$

2. LaPlace versions of the mass balance?

$$GLU = \frac{s + \alpha}{s^2 + s(\alpha + \delta) + \alpha\delta + \beta\gamma}\dot{m}_{inGLU} - \frac{\gamma}{s^2 + s(\alpha + \delta) + \alpha\delta + \beta\gamma}\dot{m}_{inINS}$$

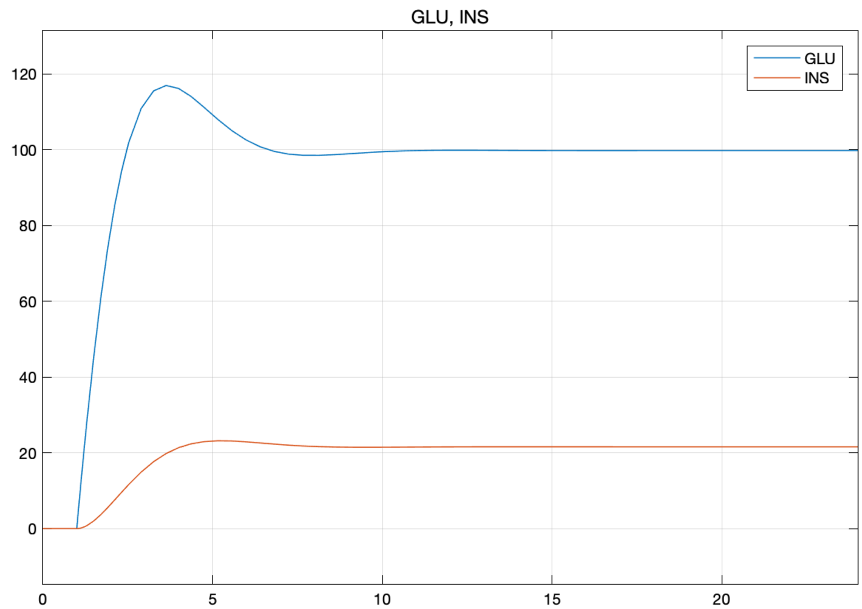
and

$$INS = \frac{\beta}{s^2 + s(\alpha + \delta) + \alpha\delta + \beta\gamma}\dot{m}_{inGLU} + \frac{s + \delta}{s^2 + s(\alpha + \delta) + \alpha\delta + \beta\gamma}\dot{m}_{inINS}$$

3. From visual inspection of the transfer functions from #2, what is the order of each system?

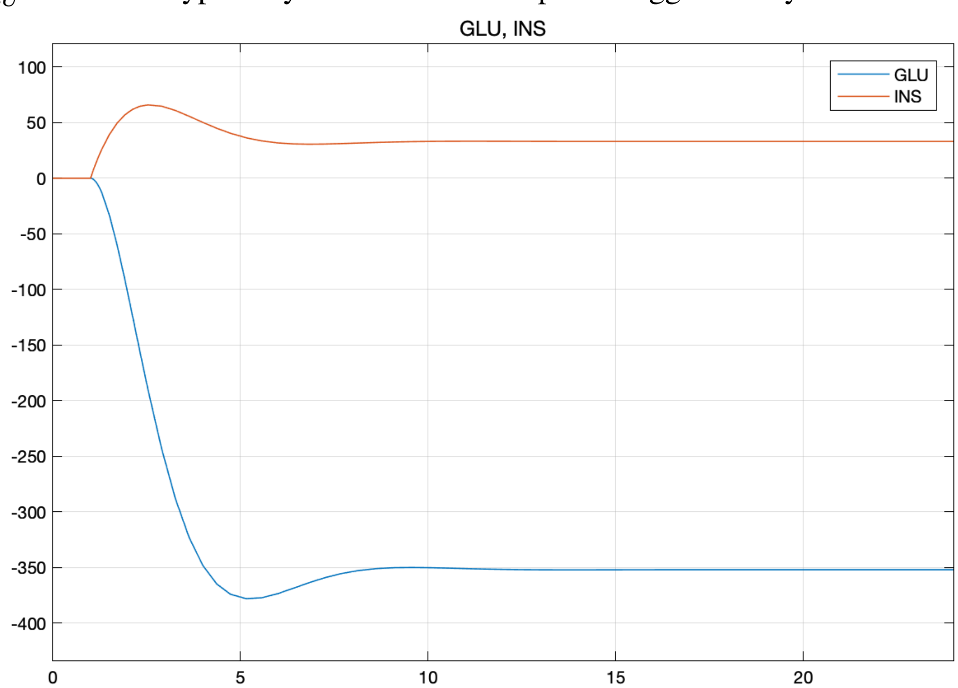
second order from the polynomial in the denominator

4. Use a SCOPE block to plot GLU and INS if $\dot{m}_{in\,GLU}$ is a step input with a value of 100 gm/hr and $\dot{m}_{in\,INS} = 0$. What type of system does each response suggest? Why?



second order; overshoot on both curves

5. Plot GLU and INS if $\dot{m}_{in\,INS}$ is an insulin step input with a value of 100 gm/hr and $\dot{m}_{in\,GLU} = 0$. What type of system does each response suggest? Why?



second order; overshoot on both plots

6. What is the %OS for #5 and #6? NOTE: you cannot calculate ζ using the formulas we used in class because the transfer function has the term $(s+a)$ in the numerator. Instead, write the output of each to the workspace and find the maximum and then calculate the %OS.

GLU step

GLU
116.9/99.8 = 17.1% OS

INS

23.2/21.6 = 7.4% OS

INS only

INS
65.9/33.1 = 99% OS

GLU
-377.9/-351.9 = 7.4% OS

7. Assume the glucose transfer function is $T_{GLU}(s) = \dfrac{GLU(s)}{\dot{m}_{inGLU}}$ when $\dot{m}_{inINS} = 0$. Assume $T_{INS}(s) = \dfrac{INS(s)}{\dot{m}_{inINS}}$ when $\dot{m}_{inGLU} = 0$. Are $T_{GLU}(s)$ and $T_{INS}(s)$ stable? How do you know?

$$T_{GLU}(s) = \frac{GLU}{\dot{m}_{inGLU}} = \frac{s+\alpha}{s^2 + s(\alpha+\delta) + \alpha\delta + \beta\gamma} = \frac{s + 0.916}{s^2 + 1.22s + 0.918}$$

$$T_{INS}(s) = \frac{INS}{\dot{m}_{inINS}} = \frac{s+\delta}{s^2 + s(\alpha+\delta) + \alpha\delta + \beta\gamma}$$

find roots of denom; system is stable (roots are the same in both cases, so both systems stable); roots also show that the system is underdamped

8. Determine the steady-state error for $T_{GLU}(s)$ and $T_{INS}(s)$. For $T_{GLU}(s)$ SSE, assume $\dot{m}_{inINS} = 0$ and $\dot{m}_{inGLU}$ is a step input of amplitude 100 gm/hr. For $T_{INS}(s)$ SSE, assume $\dot{m}_{inGLU} = 0$ and $\dot{m}_{inINS}$ is a step input of amplitude 100 gm/hr.

   We're already given $T(s)$ so use the formula from Lesson 17, p. 3. $R(s)$ is the input step function, or 100/s in LaPlace notation

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \left[sR(s)\left(1 - T(s)\right) \right]$$

   GLU
   SSE = 0.2183

   INS
   SSE = 66.88